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Visibility testing and counting ☆

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ABSTRACT

For a set of n disjoint line segments S in $\mathbb{R}^2$, the visibility testing problem (VTP) is to test whether the query point p sees a query segment $s \in S$. For this configuration, the visibility counting problem (VCP) is to preprocess S such that the number of visible segments in S from any query point p can be computed quickly. In this paper, we solve VTP in expected logarithmic query time using quadratic preprocessing time and space. Moreover, we propose a $(1+\delta)$-approximation algorithm for VCP using at most quadratic preprocessing time and space. The query time of this method is $O_\epsilon(\frac{1}{\delta^2}\sqrt{n})$ where $O_\epsilon(f(n)) = O(f(n)n^\epsilon)$ and $\epsilon > 0$ is an arbitrary constant number.

1. Introduction

Statement of the problem Suppose that we have a bounding box which covers a set S of n disjoint segments. Two points $p, q \in \mathbb{R}^2$ are visible from each other with respect to (w.r.t.) S, if there exists no segment $s \in S$ intersecting line segment $\overline{pq}$. We say that a segment $\overline{st} \in S$ is visible (w.r.t. S) from a point p, if a point $q \in \overline{st}$ can be found from which p is visible. Two problems are considered here: *the visibility testing problem (VTP)* in which we decide whether or not a given query segment $s \in S$ is visible to a query point p, and *the visibility counting problem (VCP)* for which we count the number of segments $s \in S$ which are visible to a given query point p. Scrutinizing the visibility polygon of the query point can be helpful for answering the mentioned problems. The visibility polygon of a given point $p \in \mathbb{R}^2$ is defined as

$$VP_S(p) = \{q \in \mathbb{R}^2 : p \text{ and } q \text{ are visible (w.r.t. } S)\},$$

and the visibility polygon of a given segment $\overline{st}$ is defined as

$$VP_S(\overline{st}) = \bigcup\nolimits_{q \in \overline{st}} VP_S(q) = \{p \in \mathbb{R}^2 : \overline{st} \text{ and } p \text{ are visible (w.r.t. } S)\}.$$

Consider the $2n$ endpoints of the segments of S as vertices of a geometric graph. Add a straight line edge between each pair of vertices that are visible to each other. The result is *the visibility graph of* S or $VG(S)$. We can extend each edge of $VG(S)$ in both directions until it intersects the segments in S (or the bounding box). Each edge in $VG(S)$ creates at most two new vertices and some new edges. Adding all these vertices and edges to $VG(S)$ results in a new graph called *the extended visibility graph* or $EVG(S)$.

Related works The optimal running time to compute $VP_S(p)$ is $O(n \log n)$ that uses $O(n)$ space [1,2]. Vegter [3]

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has presented an output sensitive algorithm in which preprocessing steps are in $O(n^2)$ time and $O(n^2)$ space and as a result, $VP_S(p)$ is computed in $O(|VP_S(p)|\log(\frac{n}{|VP_S(p)|}))$ time where $|VP_S(p)|$ is the number of vertices in $VP_S(p)$.

Gudmundsson and Morin considered a version of VTP where a segment $s \in S$ is chosen randomly in the preprocessing time and the goal is to test whether any given query point p can see s. Suppose that m_s is the number of edges of $EVG(S)$ incident on s and $m_s \le k \le m_s^2$. They gave an algorithm with preprocessing time and space of $O(k)$ that answers each query in $O_\epsilon(\frac{m_s}{\sqrt{k}})$ [4]. Also, the presented algorithm in [5,6] answers each query in $O(\log n)$ time by using $O(m_s^2/l)$, where $l \ge 1$ is a space/time tradeoff parameter of the data structure.

The version of VTP where s is fixed and chosen in the preprocessing time and VCP can be solved using $EVG(S)$. There is an optimal $O(n\log n + m)$ time algorithm to compute $VG(S)$ [7], where $m = O(n^2)$ is the number of the edges of $VG(S)$. This algorithm can be used to compute $EVG(S)$ in $O(n\log n + m)$ time as well [7]. Considering the planar arrangement of the edges of $EVG(S)$ as a planar graph, all points in any face of this arrangement have the same set of visible segments. The number of visible segments can be computed for each face in the preprocessing step. Since there are $O(n^4)$ faces in the planar arrangement of $EVG(S)$, a point location structure of size $O(n^4)$ can answer each query in $O(\log n)$ time. As can be seen, the space and the time used in the preprocessing step is high. However, without any preprocessing, the query can be answered by computing the visibility polygon of query point in $O(n\log n)$ time which is also high. There are also some other results with a trade-off between the query time and the preprocessing cost [8–12] (a complete survey is presented in the visibility book of [13]).

The primary concern in VCP is to propose an approximation algorithm with acceptable approximation factor which reduces the preprocessing cost. The first approximation algorithm for VCP was proposed by [2]. They represented the visibility polygon of each segment by using the union of a set of triangular convex regions. The approximation factor for the resulting algorithm was 3. With an improved covering scheme, Gudmundsson and Morin presented a 2-approximation algorithm [4]. Let $0 < \alpha \le 1$, using a data structure of size $O_\epsilon(m^{1+\alpha})$ and a preprocessing time of $O_\epsilon(m^{1+\alpha})$, this algorithm can answer each query in $O_\epsilon(m^{(1-\alpha)/2})$ time. If we show the number of visible segments from p by m_p, m'_p is also returned by this algorithm such that $m_p \le m'_p \le 2m_p$. Refs. [14] and [15] also provide the same result. Throughout this paper, $\epsilon > 0$ is an arbitrary small constant number and $O_\epsilon(f(n)) = O(f(n)n^\epsilon)$.

Two other approximation algorithms have also been introduced for VCP by Fischer et al. [5,6]. In the first algorithm, an (r/m)-cutting, $1 \le r \le n$, for $EVG(S)$ is built using a data structure of size $O((m/r)^2)$. With this cutting, the queries are answered in $O(\log n)$ time and an absolute error of r compared to the exact answer. In the second algorithm, a random sampling method is used to build a data structure of size $O((m^2\log^{O(1)} n)/l)$, $1 \le l \le n$, and any query is answered in $O(l\log^{O(1)} n)$ time. Note that, for any constant $\delta > 0$, we can have an approximation up to an absolute value of δn for the VCP. Moreover, δ can have effects on the constant factor of both the data structure size and the query time.

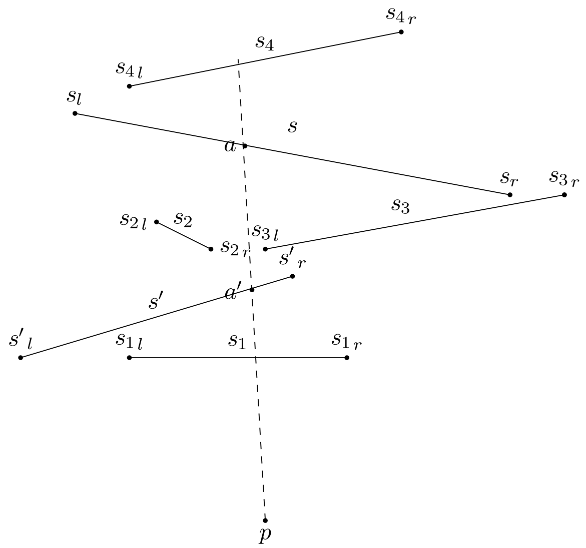

Fig. 1. $pr_p(a') = a$ and for any point $q \in s$ there is a point $q' \in s_i$, such that $s_i \in \{s', s_2, s_3\}$ and $pr_p(q') = q$.

Our results In this paper, we first propose an algorithm for VTP which answers each query in an expected $O(\log n)$ time. This algorithm uses $O(n^2)$ space and $O(n^2.\alpha(n))$ preprocessing time where $\alpha(n)$ is a pseudo-inverse of Ackermann's function. We also present a $(1+\delta)$-approximation algorithm for VCP using a randomized approach. This algorithm uses $O(n^2.\alpha(n))$ preprocessing time and $O(n^2)$ space and answers each query in $O_\epsilon(\frac{1}{\delta^2}\sqrt{n})$ time. Our experimental results demonstrate the efficiency of the algorithm.

In Section 2, we introduce our algorithm for VTP. In Section 3, the algorithm for VCP is proposed. In Section 4, we present our experimental results and finally Section 5 contains a summary of the results and the conclusion.

2. Visibility testing problem

In this section, we propose an algorithm to solve VTP. The preliminary version of this result appeared in [14]. Let $r_{\vec{d}}(p)$ denote a ray emanating from p in the direction of $\vec{d}$. Also, assuming that $s', s \in S$, by $pr_p(a') = a$ we mean that $r_{\overrightarrow{pa'}}(p)$ intersects $a \in s$ right after it intersects $a' \in s'$ (see Fig. 1).

Assume that $s \in S$ and a point p are the query inputs. If s is not visible from p, then there must exist a subset of $S - s$ such that for any point $a \in s$, there is an $a' \in s'$ such that $pr_p(a') = a$ for $s' \in S - s$, see Fig. 1. So, we start from the left endpoint of s (denoted as s_l) and move towards the right endpoint, or s_r, (the left and right endpoints of any segment are defined according to their