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Shape ellipticity based on the first Hu moment invariant

Dragiša Žunić [a], Joviša Žunić [b,∗,1]

[a] Faculty of Economics and Engineering Management, FIMEK, Novi Sad, Serbia
[b] Computer Science, University of Exeter, Exeter, UK

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ABSTRACT

In this paper we use the first Hu moment invariant to define a new ellipticity measure. The new ellipticity measure ranges over the interval $(0, 1]$ and picks the value 1 if and only if the measured shape is an ellipse. The measure is invariant with respect to translation, rotation and scaling transformations. It is straightforward and fast to compute.

1. Introduction

Shape is a characteristic of an object (like colour or texture, for example) which allows many numerical characterizations. Such numerical characteristics enable a transformation of a difficult object comparison problem onto a, much easier, vector comparison problem. Precisely, feature vectors, whose coordinates are numerically evaluated shape characteristics, corresponded to considered objects. Then, the similarity among the objects is evaluated based on the difference between their corresponding feature vectors. Obviously, such comparison is very suitable in computer supported data manipulation tasks.

Shape properties like convexity, rectangularity, compactness, etc., can be evaluated numerically in several ways. Of course, there are also shape properties with a single method for their evaluation – simply, other related methods, have not been developed yet. A need for multiple methods, to evaluate certain shape properties, is due to the fact that there is no single shape measure/method which works efficiently in all situations.

Shape descriptors, related to a particular shape property and its related measures, have a clear geometric meaning, their behaviour is well understood and could be predicted to some extent, depending on the application considered. This is always an advantage. Of course, there are also shape analysis tools which do not possess such a nice property, yet they are still in intensive use. Let us mention here the well known Hu Moment Invariants [4]. Although introduced more than 50 years ago, their behaviour on the shape domain has been completely explained/understood yet (see the recent attempts in [8] and [9]).

This paper deals with *shape ellipticity* – the descriptor which should indicate how much a given shape differs from an ellipse. This is a recurrent problem in different research areas: an early attempt is due back to 1910 [7]. Of course, several other methods for evaluating how much a shape differs from an ellipse also exist in the literature – e.g. [1–3,5]. Notice that two approaches exist. The first one says that ellipses whose axis-ratios differ are of different shapes (such an approach is used in [2]). Another approach assumes that all ellipses are of the same shape, regardless their axis-ratios (e.g. [1,5]). It is not possible to say a priori which approach is better. In some applications

∗ Corresponding author.
E-mail addresses: dragisa.zunic@fimek.edu.rs (D. Žunić), j.zunic@ex.ac.uk (J. Žunić).

[1] Also with the Mathematical Institute, Serbian Academy of Arts and Sciences, Belgrade.

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the first approach would be more appropriate, whilst in some others, the second should be given a preference. The ellipticity measure defined in this paper uses the second approach, i.e., it assigns the maximal ellipticity value to all ellipses.

2. Preliminaries

We start with definitions necessary to define a new ellipticity measure. First we define the, so-called, *geometric moments*, $m_{p,q}(S)$, of a planar region/shape S (see [6]):

$$m_{p,q}(S) = \iint_S x^p y^q \, dx\, dy. \tag{1}$$

Throughout the paper, all appearing shapes are assumed to be of unit area, i.e. $m_{0,0}(S) = 1$ will be assumed for all appearing shapes S, even not mentioned.

The moments $m_{0,0}(S)$, $m_{1,0}(S)$, and $m_{0,1}(S)$ are used to define the shape centroid [6]. Since we assume that all appearing shape have areas equal to 1, the shape centroid can be expressed as $(m_{1,0}(S), m_{0,1}(S))$.

The new ellipticity measure, will be derived by using the first Hu moment invariant $\mathcal{I}(S)$, which is defined, [4], as:

$$\mathcal{I}(S) = \iint_S (x^2 + y^2)\, dx\, dy = m_{2,0}(S) + m_{0,2}(S). \tag{2}$$

Prior to computing their ellipticity, all shapes will be placed in the normalized position. More precisely, every shape considered will be translated so that its centroid coincides with the origin, and rotated so that its orientation (computed by the standard method [10]) becomes horizontal (i.e. orientation becomes 0 degrees). Just as a reminder, the shape orientation is used in many image preprocessing tasks as a part of image normalization procedure. The most standard method [6,10] defines the shape orientation by the line which minimizes the integral of the squared distances of shape's points to this line. The computation of such line is easy and straightforward [10]. Let us recall here that the orientation $\mathcal{O}(S)$, of S, satisfies the equation $\tan(2 \cdot \mathcal{O}(S)) = \frac{2 \cdot m_{1,1}(S)}{m_{2,0}(S) - m_{0,2}(S)}$.

It is worth mentioning that moment invariants have already been used to measure shape ellipticity. Affine invariant $\mathcal{J}(S)$, defined as:

$$\mathcal{J}(S) = \big(m_{2,0}(S) \cdot m_{0,2}(S) - m_{1,1}(S)^2\big)/m_{0,0}(S)^4 \tag{3}$$

has been used in [5] to define the ellipticity measure

$$\mathcal{E}_I(S) = \min\big\{16\pi^2 \mathcal{J}(S),\ \big(16\pi^2 \mathcal{J}(S)\big)^{-1}\big\}. \tag{4}$$

The same article presents the triangularity measure as well, which is also based on $\mathcal{J}(S)$. Both ellipticity and triangularity measures, from [5], are set to range over $[0, 1]$ and peaking at 1 for a perfect ellipse (perfect triangle). The problem is that, for both measures, if the measured ellipticity (triangularity) equals 1, it is not guaranteed (or at least not proved) that the considered shape is a perfect ellipse (triangle). The ellipticity measure defined here does not have such a disadvantage – it equals 1 if and only if the measured shape is an ellipse.

As mentioned, prior to computing the ellipticity of a given shape S, some preprocessing/normalization of S will be done. The shape obtained will be called the *normalized* shape, and it is defined as follows.

Definition 1. Let a shape S be given. S is said to be *normalized* if:

(a) The area of S is 1;
(b) The centroid of S coincides with the origin;
(c) The orientation $\mathcal{O}(S)$ of S is 0.

Finally, we define two parameters, $a(S)$ and $b(S)$, needed to define the ellipticity of S:

$$a(S) = \sqrt{2\pi^2 \mathcal{I}(S) - \pi \cdot \sqrt{4\pi^2 \mathcal{I}(S)^2 - 1}}$$
$$b(S) = \sqrt{2\pi^2 \mathcal{I}(S) + \pi \cdot \sqrt{4\pi^2 \mathcal{I}(S)^2 - 1}}. \tag{5}$$

3. The main result

Now we are prepared to give the main result/statement of the manuscript.

Theorem 1. *Let a normalized shape S be given. Then*:

(a) $a(S)^2 \cdot m_{2,0}(S) + b(S)^2 \cdot m_{0,2}(S) \geqslant 1/2$;
(b) $a(S)^2 \cdot m_{2,0}(S) + b(S)^2 \cdot m_{0,2}(S) = 1/2 \Leftrightarrow S$ *is an ellipse.*

Proof. First, we define $E(S) = \{(x, y) \mid a(S)^2 \cdot x^2 + b(S)^2 \cdot y^2 = 1\}$, as an auxiliary ellipse.

Since it is easy to verify that the equality $a(S)^2 \cdot m_{2,0}(S) + b(S)^2 \cdot m_{0,2}(S) = 1/2$ is true for any normalized ellipse S, it remains to prove that if S is not an ellipse then the inequality in (a) is strict – i.e. if S is not an ellipse than the equality in (b) cannot be true.

Let a shape S, different from an ellipse, be given. Then S differs from $E(S)$ as well, and the set differences $E(S) \setminus S$ and $S \setminus E(S)$ have the same, strictly positive, area $\Delta(S)$:

$$\Delta(S) = \text{Area\_of\_}\big(E(S) \setminus S\big) = \text{Area\_of\_}\big(S \setminus E(S)\big) > 0.$$

Next,

$$\begin{aligned}
&\big(a(S)^2 m_{2,0}(S) + b(S)^2 m_{0,2}(S)\big) \\
&\quad - \big(a(S)^2 m_{2,0}\big(E(S)\big) + b(S)^2 m_{0,2}\big(E(S)\big)\big) \\
&= \iint_S \big(a(S)^2 x^2 + b(S)^2 y^2\big)\, dx\, dy \\
&\quad - \iint_{E(S)} \big(a(S)^2 x^2 + b(S)^2 y^2\big)\, dx\, dy \\
&= \iint_{S \setminus E(S)} \big(a(S)^2 x^2 + b(S)^2 y^2\big)\, dx\, dy \\
&\quad - \iint_{E(S) \setminus S} \big(a(S)^2 x^2 + b(S)^2 y^2\big)\, dx\, dy \\
&= \Delta(S)\big(a(S)^2 u^2 + b(S)^2 v^2\big) \\
&\quad - \Delta(S)\big(a(S)^2 l^2 + b(S)^2 t^2\big).
\end{aligned} \tag{6}$$