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Embedding even cycles on folded hypercubes with conditional faulty edges

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A B S T R A C T

Let FF_e be the set of $|FF_e| \leq 2n-4$ faulty edges in an n-dimensional folded hypercube FQ_n such that each vertex in FQ_n is incident to at least two fault-free edges. Under this assumption, we show that every edge of $FQ_n - FF_e$ lies on a fault-free cycle of every even length from 6 to 2^n, where $n \geq 5$.

1. Introduction

The n-dimensional hypercube, denoted by Q_n, is one of the most versatile, efficient interconnection network [1,6]. One of the hypercube's variations is the *folded hypercube* [4], denoted by FQ_n, which is constructed by adding a link to every pair of nodes that have complementary addresses.

An *embedding* of one guest graph G into another host graph H is a one-to-one mapping f from the node set of G to the node set of H [6]. The *distance* between any two vertices, u and v, of G, denoted by $d_G(x, y)$, is the length of the shortest path between u and v. Let F_e (respectively, FF_e) be the set of faulty edges in Q_n (respectively, FQ_n). Under the *conditional fault model*, i.e., each vertex is incident to at least two fault-free edges, Tsai et al. [9] showed that each edge of $Q_n - F_e$ lies on a fault-free cycle of every even length from 6 to 2^n for $n \geq 3$, where $|F_e| \leq 2n-5$. Let F be a set of $2n-5$ faulty edges in Q_n ($n \geq 3$). Kueng et al. [5] proved that a conditionally faulty Q_n with $|F| = 2n-5$ faulty edges contains a fault-free path of length ℓ between any two nodes u and v of distance $d^* = d_{Q_n - F}(u, v) \geq 2$ for each ℓ satisfying $d^* \leq \ell \leq 2^n - 1$ and $2 \mid (\ell - d^*)$. Given a faulty FQ_n with $|FF_e| \leq n-1$ faulty edges, Xu et al. [10] proved that every fault-free edge lies on a fault-free cycle of every even length from 4 to 2^n. As to embedding odd cycles in folded hypercubes with $|FF_e| \leq 2n-5$ faulty edges under conditional fault model can be found in [3]. In this paper, we consider the FQ_n ($n \geq 3$) with $|FF_e| \leq 2n-4$ faulty edges under the conditional fault model and prove that every fault-free edge lies on a fault-free cycle of every even length from 6 to 2^n. As to embedding even cycles in FQ_n, our result improves the result by Xu et al. [10] in terms of the number of fault-tolerant edges.

2. Definitions and terminology

Let networks discussed in this paper be represented by simple undirected graphs. For graph-theoretical terminology and notation not defined here, we follow [2]. A *path* $P = \langle v_0, v_1, \cdots, v_n \rangle$ is a sequence of distinct vertices in which any two consecutive vertices are adjacent.

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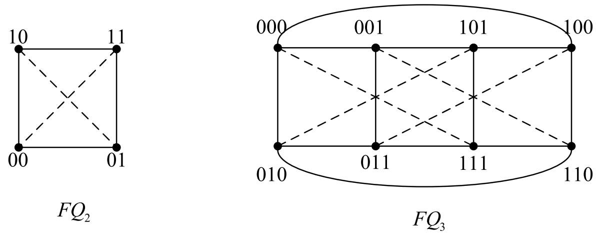

Fig. 1. FQ_2 and FQ_3, in which the dashed lines represent the complementary edges.

P is also denoted by $P = \langle v_0, \cdots, v_i, P[v_i, v_j], v_j, \cdots, v_n\rangle$, where $P[v_i, v_j] = \langle v_i, v_{i+1}, \cdots, v_{j-1}, v_j\rangle$ if it exists. Two paths P_1 and P_2 are *vertex-disjoint* if and only if $V(P_1) \cap V(P_2) = \emptyset$. A cycle is a path with at least three vertices, where the two end-vertices are adjacent. The *length* of a path P (respectively, cycle C), denoted by $\ell(P)$ (respectively, $\ell(C)$), is the number of the edges in P (respectively, C). For a graph G, G is *edge-bipancyclic* if each edge lies on a cycle of every even length from 4 to $|V(G)|$. An *isomorphism* from a graph G to a graph H is a bijection $\pi : V(G) \to V(H)$ such that $(u, v) \in E(G)$ if and only if $(\pi(u), \pi(v)) \in E(H)$. G is called isomorphism to H, denoted by $G \cong H$. An *automorphism* of G is an isomorphism from G to G. A graph G is *edge-transitive* if for any two edges e_1 and e_2 in $E(G)$, there exists an automorphism that maps e_1 to e_2.

An n-dimensional hypercube Q_n is represented by $Q_n = (V(Q_n), E(Q_n))$, where $V(Q_n) = \{u_n u_{n-1} \ldots u_1 \mid u_i \in \{0, 1\}$ for $i = 1, 2, \cdots, n\}$. $(u, v) \in E(Q_n)$ if and only if u and v differ in exactly one bit of their labels. Let $E_k = \{(u_n u_{n-1} \ldots u_{k+1} 0 u_{k-1} \ldots u_1, u_n u_{n-1} \ldots u_{k+1} 1 u_{k-1} \ldots u_1) \mid u_i \in \{0, 1\}$ for $1 \le i \le n$ and $i \ne k\}$. An edge in E_k is called *along dimension* k. Q_n can be partitioned along dimension k ($1 \le k \le n$) into two subcubes $Q^{k,0}_{n-1}$ and $Q^{k,1}_{n-1}$, where $Q^{k,0}_{n-1}$ is induced by the vertex set $\{u_n u_{n-1} \ldots u_{k+1} 0 u_{k-1} \ldots u_1 \mid u_i \in \{0, 1\}$ for $1 \le i \le n$ and $i \ne k\}$ and $Q^{k,1}_{n-1}$ is induced by the vertex set $\{u_n u_{n-1} \ldots u_{k+1} 1 u_{k-1} \ldots u_1 \mid u_i \in \{0, 1\}$ for $1 \le i \le n$ and $i \ne k\}$. The edges between $Q^{k,0}_{n-1}$ and $Q^{k,1}_{n-1}$ are called *crossing edges*.

An n-dimensional folded hypercube FQ_n is constructed from Q_n by adding an edge, called *complementary edge*, between every two vertices with complementary addresses (i.e., vertex $x = x_n x_{n-1} \ldots x_1$ and vertex $\bar{x} = \bar{x}_n \bar{x}_{n-1} \ldots \bar{x}_1$). FQ_2 and FQ_3 are shown in Fig. 1. We use E_a to denote the set of all complementary edges, then FQ_n can be represented by $FQ_n = Q_n \cup E_a$. It has been shown that FQ_n is $(n + 1)$-regular, $(n + 1)$-connected, vertex-transitive and edge-transitive [10].

3. Main results

In this section, we will prove our main result. Firstly, we need the following lemmas.

Lemma 1. *(See [10].) $FQ_n - E_i \cong Q_n$ for each $i \in \{1, 2, \cdots, n, a\}$.*

By Lemma 1, $FQ_n - E_i \cong Q_n$ and Q_n can be partitioned along dimension k into two subcubes $Q^{k,0}_{n-1}$ and $Q^{k,1}_{n-1}$ (abbreviated as Q^0_{n-1} and Q^1_{n-1} if k is specified). The vertex with labels $0u_{n-1} \ldots u_1$ (respectively, $1u_{n-1} \ldots u_1$) of FQ_n in Q^0_{n-1} (respectively, Q^1_{n-1}) is denoted by $0u$ (respectively, $1u$), i.e., $0u = 0u_{n-1} \ldots u_1$ (respectively, $1u = 1u_{n-1} \ldots u_1$). The crossing edge containing $0u$ is denoted by $(0u, 1u)$ and the complementary edge containing $0u$ is denoted by $(0u, 1\bar{u})$, where $1u = 1u_{n-1} \ldots u_1$ and $1\bar{u} = 1\bar{u}_{n-1} \ldots \bar{u}_1$.

Lemma 2. *(See [10].) Let F_e be a set of $n - 2$ faulty edges in Q_n ($n \ge 2$). Suppose that u and v are any two different vertices of $Q_n - F_e$. Then $Q_n - F_e$ contains a path of length ℓ between u and v for every ℓ satisfying $d_{Q_n - F_e}(u, v) \le \ell \le 2^n - 1$ and $2 \mid (\ell - d_{Q_n - F_e}(u, v))$.*

Lemma 3. *(See [7].) If $n \ge 2$, then every edge of Q_n lies on a cycle of every even length from 4 to 2^n.*

Lemma 4. *(See [8].) Assume that $n \ge 2$. Let X and Y be any two partite sets of Q_n. In addition, x and u are any two distinct vertices of X; and y and v are any two distinct vertices of Y. Then there exist two vertex-disjoint paths P_1 and P_2 such that: (1) P_1 connects u to v, (2) P_2 connects x to y, and (3) $V(P_1) \cup V(P_2) = V(Q_n)$.*

Lemma 5. *(See [9].) Every fault-free edge in $Q_n - F_e$ lies on a fault-free cycle of every even length from 6 to 2^n under the condition that $|F_e| \le 2n - 5$ and each vertex is incident to at least two fault-free edges, where $n \ge 3$.*

We present our main result as follows.

Theorem 1. *Let FF_e be the set of faulty edges in FQ_n for $n \ge 5$. Assume that $|FF_e| \le 2n - 4$ and each vertex in FQ_n is incident to at least two fault-free edges, then every edge of $FQ_n - FF_e$ lies on a fault-free cycle of every even length from 6 to 2^n.*

Proof. Since each vertex in FQ_n is incident to at least two fault-free edges. If $|FF_e| \le 2n - 5$, then let $k = (2n - 4) - |FF_e|$. Let $M_1 = \{e_1, e_2, \ldots, e_k\}$ and $M_2 = \{e'_1, e'_2, \ldots, e'_k\}$ be two sets of fault-free edges such that each vertex in $FQ_n - FF_e - M_1$ (respectively, $FQ_n - FF_e - M_2$) is incident to at least two fault-free edges, where $M_1 \cap M_2 = \emptyset$ and $FQ_n - FF_e = (FQ_n - FF_e - M_1) \cup (FQ_n - FF_e - M_2)$. (Since $|E(FQ_n)| = 2^{n-1}(n + 1) > 2(2n - 4) \ge 2k$, M_1 and M_2 indeed exist.) Let $FF^i_e = FF_e \cup M_i$ for $i = 1, 2$. Then $|FF^i_e| = 2n - 4$. Hence, if $|FF_e| \le 2n - 5$, by taking the edges in M_i ($i = 1, 2$) as fault-free edges temporarily, we need to prove that each edge in $FQ_n - FF^i_e$ lies on a fault-free cycle of every even length from 6 to 2^n, where $|FF^i_e| = 2n - 4$. If $|FF_e| = 2n - 4$, we need to prove that each edge in $FQ_n - FF_e$ lies on a fault-free cycle of every even length from 6 to 2^n. By the same number of faulty edges in FF^i_e (when $|FF_e| \le 2n - 5$) as that in FF_e (when $|FF_e| = 2n - 4$), in the following, we only need to consider $|FF_e| = 2n - 4$.

Consider any fault-free edge e in FQ_n. From $E(FQ_n) = \cup^n_{i=1} E_i \cup E_a$, we can see that there exists an $i \in \{1, 2, \ldots, n, a\}$ such that $e \in E_i$. Since FQ_n is edge-transitive, we may