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On the partition dimension of a class of circulant graphs

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ABSTRACT

For a vertex v of a connected graph $G(V,E)$ and a subset S of V, the distance between a vertex v and S is defined by $d(v,S) = \min\{d(v,x)\colon x \in S\}$. For an ordered k-partition $\pi = \{S_1, S_2 \ldots S_k\}$ of V, the partition representation of v with respect to π is the k-vector $r(v|\pi) = (d(v,S_1), d(v,S_2) \ldots d(v,S_k))$. The k-partition π is a resolving partition if the k-vectors $r(v|\pi)$, $v \in V(G)$ are distinct. The minimum k for which there is a resolving k-partition of V is the partition dimension of G. Salman et al. [1] in their paper which appeared in *Acta Mathematica Sinica, English Series* proved that partition dimension of a class of circulant graph $G(n, \pm\{1,2\})$, for all even $n \geqslant 6$ is four. In this paper we prove that it is three.

1. Introduction

The concepts of resolvability and location in graphs were described independently by Slater [2] and Harary and Melter [3], to define the same structure in a graph. After these papers were published several authors developed diverse theoretical works about this topic [3–8]. Slater described the usefulness of these ideas into long range aids to navigation [2]. Also, these concepts have some applications in chemistry for representing chemical compounds [9] and to problems of pattern recognition and image processing, some of which involve the use of hierarchical data structures. Other applications of this concept to navigation of robots in networks and other areas appear in [4,10]. Some variations on resolvability or location have been appearing in the literature, like those about conditional resolvability [11], locating domination [12], resolving domination [13] and resolving partitions [4,14–16].

Given a graph $G = (V,E)$ and an ordered set of vertices $S = \{v_1, v_2 \ldots v_k\}$ of G, the representation of a vertex $v \in V$ with respect to the set S is the vector $r(v|S) = (d(v,v_1), d(v,v_2) \ldots d(v,v_k))$, where $d(v,v_i)$ denotes the distance between the vertices v and v_i, $1 \leqslant i \leqslant k$. We say that S is a *resolving set* if different vertices of G have different representations, i.e., for every pair of vertices $u, v \in V$, $r(u|S) \neq r(v|S)$. The *metric dimension* of G is the minimum cardinality of any resolving set of G, and it is denoted by $\dim(G)$.

Given an ordered partition $\pi = \{S_1, S_2 \ldots S_k\}$ of the vertices of G, the partition representation of a vertex $v \in V$ with respect to the given partition π is the vector $r(v|\pi) = (d(v,S_1), d(v,S_2) \ldots d(v,S_k))$ where $d(v,S_i)$, $1 \leqslant i \leqslant k$, represents the distance between the vertex v and the set S_i, i.e., $d(v,S_i) = \min\{d(v,u)\colon u \in S_i\}$. We say that π is a resolving partition if different vertices of G have different partition representations, i.e., for every pair of vertices $u, v \in V$, $r(u|\pi) \neq r(v|\pi)$. The *partition*

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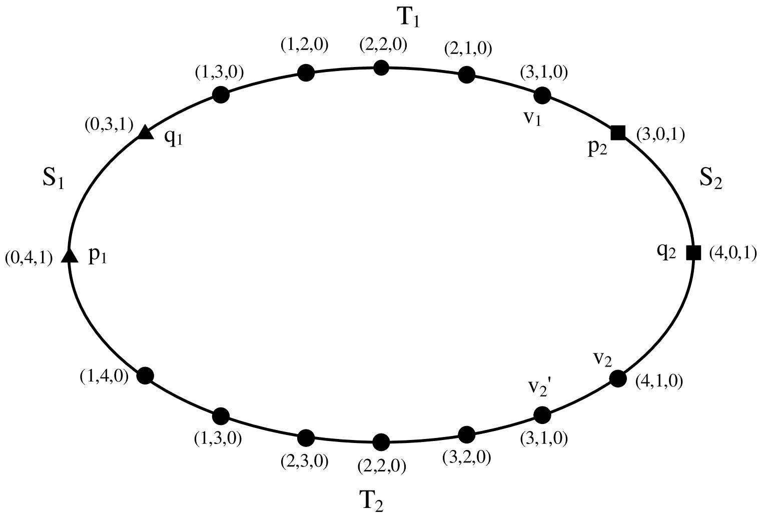

Fig. 1. Case when $|T_2| = |T_1| + 2$ in $G(16, \pm\{1, 2\})$ with $r = 3$.

dimension of G is the minimum number of sets in any resolving partition for G and it is denoted by $pd(G)$. The *partition dimension* of graphs is studied in [6,14,15,17].

It is natural to think that the *partition dimension* and *metric dimension* are related; in [14] it was shown that for any nontrivial connected graph G, $pd(G) \leqslant \dim(G) + 1$. It is also shown that $pd(G) = 2$ if and only if $G = P_n$ [14].

2. Circulant graphs

The circulant graph is a natural generalization of the double loop network and was first considered by Wong and Coppersmith [18]. Circulant graphs have been used for decades in the design of computer and telecommunication networks due to their optimal fault-tolerance and routing capabilities [19]. It is also used in VLSI design and distributed computation [20–22]. The term circulant comes from the nature of its adjacency matrix. A matrix is circulant if all its rows are periodic rotations of the first one. Circulant matrices have been employed for designing binary codes [23]. Theoretical properties of circulant graphs have been studied extensively and surveyed by Bermond et al. [20]. Every circulant graph is a vertex transitive graph and a Cayley graph [24]. Most of the earlier research concentrated on using the circulant graphs to build interconnection networks for distributed and parallel systems [19, 20].

An undirected circulant graph, denoted by $G(n, \pm\{1, 2 \ldots j\})$, $1 \leqslant j \leqslant \lfloor n/2 \rfloor$, $n \geqslant 3$ is defined as a graph consisting of the vertex set $V = \{0, 1 \ldots n - 1\}$ and the edge set $E = \{(i, j)\colon\ |j - i| \equiv s \pmod{n},\ s \in \{1, 2 \ldots j\}\}$. It is also clear that $G(n, \pm 1)$ is an undirected cycle and $G(n, \pm\{1, 2 \ldots \lfloor n/2 \rfloor\})$ is the complete graph K_n. We observe that $G(n, \pm\{1\})$ is a subgraph of $G(n, \pm\{1, 2 \ldots j\})$ for every j, $1 \leqslant j \leqslant \lfloor n/2 \rfloor$.

The following theorem is due to Salman et al.

Theorem 2.1. *(See [1].) For a family of circulant graphs $G(n, \pm\{1, 2\})$, $pd(G) = 4$ for all even $n \geqslant 6$ and $n = 7$.*

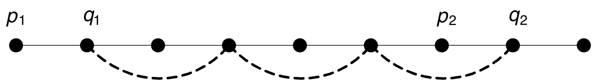

Fig. 2. Path from q_1 to S_2.

It is known that the $\dim(G(n, \pm\{1, 2\})) = 3$, when $n \equiv 0, 2$ or $3 \pmod 4$ and $2 < \dim(G(n, \pm\{1, 2\})) \leqslant 4$, when $n \equiv 1 \pmod 4$ [25].

In this paper we consider the class of circulant graphs $G = G(n, \pm\{1, 2\})$, $n \equiv 0 \pmod 4$, $n \geqslant 12$, and prove that the partition dimension is three, thereby improving Theorem 2.1.

3. Main results

Salman et al. [1] have proved that Theorem 2.1 for all even $n \geqslant 6$. We disprove this result for all even $n \geqslant 12$, $n \equiv 0 \pmod 4$. In order to obtain a resolving partition, we require the following preliminaries.

Let $G = G(n, \pm\{1, 2\})$, $n \equiv 0 \pmod 4$, $n \geqslant 12$. Let T_1 be any set of consecutive vertices of G. Let S_1 and S_2 be the sets of vertices at distance one from T_1. Put $T_2 = V(G) - (S_1 \cup S_2 \cup T_1)$ and $S_3 = T_1 \cup T_2$. Now S_1 and S_2 are 2 subsets of V. So let $S_1 = \{p_1, q_1\}$ and $S_2 = \{p_2, q_2\}$. Also assume that p_1, q_1, p_2, q_2 lie on the outer cycle of G in the clockwise direction.

Lemma 3.1. *Let $G = G(n, \pm\{1, 2\})$, $n \equiv 0 \pmod 4$, $n \geqslant 12$. Then $\pi = \{S_1, S_2, S_3\}$ is not a resolving partition when*

(1) *$|T_1|$ or $|T_2|$ is even.*
(2) *$|T_2| \sim |T_1| = 2$.*
(3) *$|T_2| \sim |T_1| = 4m$, $1 \leqslant m \leqslant 1 + (n - 12)/4$.*

Proof.

(1) Let $|T_1|$ be even. Without loss of generality, let $T_1 = \{0, 1 \ldots 2k - 1\}$ for some positive integer k. In this case $S_1 = \{n - 2, n - 1\}$ and $S_2 = \{2k, 2k + 1\}$. The vertices 0