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Inefficiency analysis of the scheduling game on limited identical machines with activation costs [☆]

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ABSTRACT

We investigate the scheduling game on a fixed number m of identical machines that no machines are initially activated and each machine activated incurs the same activation cost. Every job, as a selfish player, is interested in minimizing its own individual cost composing of both the load of its chosen machine and its share in the machine's activation cost, whereas the social cost focuses on the sum of makespan and total activation cost. The inefficiency of pure Nash equilibria is assessed by the Price of Anarchy (PoA) and Price of Stability (PoS). First, when the jobs' total length is no larger than a single machine's activation cost, we demonstrate that m is the tightness upper bound of PoA and PoS equals to 1. Then, for the case that the total length is large, we prove that the PoA is tightly bounded by $(m+1)/2$. Finally, a lower bound of the PoS is also given.

1. Introduction

In the classical machine scheduling problems, a fixed set of machines is always provided initially and all of them can be utilized freely. Motivated by real machines should be paid before using and the performance of scheduling can be highly dependent on the number of machines, Imreh and Noga [1] proposed the concept of machine cost. With the objective of minimizing the sum of the makespan and cost of machines activated, online or semi-online algorithms have been extensively studied in the literature [2–4] under the assumption of sufficient identical machines available. In addition, when only two uniform machines can be activated, Han et al. [5] designed optimal online algorithms for solving this problem.

In the last decade, game theory has been incorporated into many combinatorial optimization problems and received wide attention. Take machine scheduling for example, each job treated as a game a player will choose a machine to be processed on as it's strategy instead of being controlled by a central designer. The social optimum may not be typically obtained as the players act selfishly until reaching some Nash Equilibrium (NE). Quantifying the loss due to selfish behaviors becomes crucial. The Price of Anarchy (PoA) was first proposed by Koutsoupias and Papadimitriou [6] to measure the inefficiency of equilibria. It has been widely investigated in the literature [7–9].

Recently, Feldman and Tamir [10] developed a game scheduling model with machine activation cost, in which a job's individual cost is composed of both its machine's load and its proportionally shared activation cost. While there are unlimited number of identical machines provided, the inefficiency of equilibria is measured with a social objective of minimizing the maximum individual cost.

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Denote B and p_{max} as the activation cost and length of the longest job, respectively, they get $PoA \leq \frac{1+\alpha}{2\sqrt{\alpha}}$ and $PoS = \frac{5}{4}$ where $\alpha = \frac{B}{p_{max}}$. Under the same scenario, Chen and Gürel [11] analyzed the quality of NE with a social cost which is the sum of all jobs' costs. Here, $PoA \leq \frac{1}{2}(\rho+1)$ and $PoS \leq \frac{1}{4}(\sqrt{\rho+2})$ where $\rho = \frac{B}{p_{min}}$ and p_{min} is the length of the shortest job. When the social cost is the makespan plus total machines' activation cost, Chen and Gürel [12] also proved that $PoA \leq \frac{1}{2}\sqrt{\frac{P}{p_{min}}}$ and $PoS \leq \sqrt{\frac{P}{B}}$, in which P denotes the total jobs' length. To the best of our knowledge, although the pure NE existence has also been proved in [10] if there are only limited identical machines can be activated, the inefficiency analysis has never been discussed. We study this game scheduling problem with the social objective of minimizing the sum of makespan and total activation cost.

The remainder of this paper is organized as follows. In Section 2, we describe the model in detail and give some preliminaries. In Section 3, the upper bounds of PoA and PoS are derived when the total length is no larger than a single machine's activation cost. In Section 4, we show that the PoA can be smaller if the jobs' total length is larger than the activation cost. An example is provided to illustrate that the obtained bound is tight. Meanwhile, a lower bound of the PoS is also proved through an example provided.

2. Problem description, notation and preliminaries

The problem considered in this paper can be formally characterized as follows: Given a set of limited identical machines $M = \{M_1, M_2, \dots, M_m\}$ where $m \geq 2$ and a set of jobs $J = \{J_1, J_2, \dots, J_n\}$ satisfying $m \leq n$. Here, $n < m$ is not considered as it is equivalent to the case of unlimited machines available. Each job J_j has a processing time p_j. Denote P as the total length of all jobs. Since all machines are identical, let the cost of activating each machine equal to B. Without loss of generality, we assume $B = 1$ in the remainder of our paper (this can be easily achieved by dividing all jobs' sizes with the activation cost B). Then, jobs can be divided into two categories, large and small: $J_{la} = \{J_j \in J : p_j > 1\}$ and $J_{sm} = \{J_j \in J : p_j \leq 1\}$. Similarly, if $P > 1$, we say the total length is large, otherwise it's small. A schedule is denoted as a vector $s = (s_1, s_2, \dots, s_n)$ which means the jth job is processed on $s_j \in M$. Given an overall schedule s, the individual cost of job j is defined as follows:

$$IC(j) = L_i^s + \frac{p_j}{L_i^s},$$

where $L_i^s = \sum_{s_j = M_i} p_j$ is the load of machine M_i and the second part represents job j's share of the activation cost, which is proportional to its length. From the social perspective, the makespan of all activated machines as well as the total activation cost are all important. Therefore, the social cost function of s is as follows:

$$SC(s) = m_s + L_{\max}^s,$$

where m_s and $L_{\max}^s$ are the number of machines activated in s and the makespan, respectively.

Let S denote the set of all schedules for the problem instance (J, M). Denote $OPT(J, M) = \min_{s \in S} SC(s)$ as the optimal social cost. A schedule $s \in S$ is a pure NE if no $j \in J$ can benefit from unilaterally deviating from its machine to another machine. $\phi(J, M)$ denotes the set of all NE schedules of the instance (J, M). m^* and m_{NE} are the number of machines activated in the optimal schedule and a NE schedule, respectively. The number of jobs processed on M_i is denoted as n_i.

Combining with the above model, we give the following definition of PoA and PoS.

Definition 1. If $\phi(G) \neq \emptyset$, the PoA is the ratio between the social cost of the worst NE schedule and the social optimum, that is,

$$PoA = \sup_{(J,M)} \frac{\max_{s \in \phi(J,M)} SC(s)}{OPT(J, M)}.$$

The PoS is the ratio between the social cost of the best NE schedule and the social optimum, that is,

$$PoS = \sup_{(J,M)} \frac{\min_{s \in \phi(J,M)} SC(s)}{OPT(J, M)}.$$

For notational convenience, we may omit (J, M) and the superscript s if there is no confusion. Next, some lemmas which will be used in the subsequent sections are presented.

Lemma 1. *(See [11].) In any NE assignment, if $m_{NE} < m$, then any large job will be assigned to a dedicated machine, and for machines of small jobs $L_i \leq 1$.*

Lemma 2. *If only a limited number of machines are available, then in any NE schedule, the difference in load between the most-loaded machine which at least a small job has been processed on and any other machine is no more than 1.*

Proof. Suppose to the contrary that M_i is a machine satisfying $L_{max} - L_i > 1$ in a NE assignment s, then we can get $L_{max} > 1$. Moving any small job j from the most-loaded machine to M_i will result in a reduced individual cost $IC'(j)$:

$$\begin{aligned} IC(j) - IC'(j) &= L_{max} + \frac{p_j}{L_{max}} - (L_i + p_j + \frac{p_j}{L_i + p_j}) \\ &= (L_{max} - L_i - p_j)(1 - \frac{p_j}{L_{max}(L_i + p_j)}) \\ &> 0, \end{aligned}$$

which contradicts with the NE assignment. Hence, the lemma holds. □

3. PoA and PoS with respect to a small total length

In this section, we will prove the upper bounds of PoA and PoS under the condition of $P \leq 1$. That is to say, all jobs are small and the total jobs' length is no larger than the activation cost $B = 1$.