Information Processing Letters 114 (2014) 130–136

Contents lists available at ScienceDirect

Information Processing Letters

www.elsevier.com/locate/ipl

Optimum sweeps of simple polygons with two guards

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ARTICLE INFO

Article history:
Received 15 March 2013
Received in revised form 30 October 2013
Accepted 1 November 2013
Available online 6 November 2013
Communicated by R. Uehara

Keywords:
Computational geometry
Visibility
Ray shooting
Polygon sweep problem
Two-guard problem

ABSTRACT

A polygon P admits a *sweep* if two mobile guards can detect an unpredictable, moving target inside P, no matter how fast the target moves. Two guards move on the polygon boundary and are required to always be mutually visible. The objective of this study is to find an optimum sweep such that the sum of the distances travelled by the two guards in the sweep is minimized. We present an $O(n^2)$ time and $O(n)$ space algorithm for optimizing this metric, where n is the number of vertices of the given polygon. Our result is obtained by reducing this problem to finding a shortest path between two nodes in a graph of size $O(n)$.

1. Introduction

Motivated by the relations to the well-known *Art Gallery* and *Watchman Route* problems, much attention has recently been devoted to the problem of detecting an unpredictable, moving target in an n-sided polygon P by a group of mobile guards. Both the target and the guards are modeled by points that can continuously move in P. The goal of the guards is to eventually "see" the target, or to verify that no target is present in the polygon, no matter how fast the target moves. Many types of polygon shapes and the visibilities of the guards have been studied in the literature [3–8,10,11].

In this paper, we focus on the two-guard model studied in [3,6], in which two (point) guards move on the polygon boundary, always remaining mutually visible. The goal is to sweep P with two guards so that at any instant, the line segment connecting the guards partitions P into a "cleared" region (not containing the target) and an "unexplored" region (it may contain the target). In the end, we would like to know whether the whole polygon P is cleared or the target is detected, if it is ever possible. This target-finding model has obvious advantages for safety and communication between the guards.

Icking and Klein were the first to study the problem of sweeping corridors with two guards, which was called the *two-guard problem* [6]. A simple polygon with an *entrance* s and an *exit* t on its boundary is called a *corridor*. Two guards move on the boundary of the corridor P, starting at s, and finally force the target out of P through t. They gave an $O(n \log n)$ time algorithm for determining whether a corridor can be swept with two guards [6]. Later, a linear-time algorithm was presented by Heffernan [5]. Tseng et al. gave an $O(n \log n)$ time algorithm to determine whether there is a pair of vertices in P that allows a sweep. This result has also been improved to $O(n)$ [1].

The problem of sweeping simple polygons with two guards was further studied in [10,11]. Since neither the entrance nor the exit on the polygon boundary is given, the starting point (on the polygon boundary) of any sweep schedule may be visited by the target for the second or more time, i.e., *recontamination* is generally necessary for the problem of sweeping simple polygons with a chain of guards [3,4]. This makes it more difficult and challenging. A linear-time algorithm has been presented for determining whether a polygon can be swept with two guards by checking on several forbidden configurations [11].

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0020-0190/$ – see front matter
http://dx.doi.org/10.1016/j.ipl.2013.11.002

An algorithm for finding the minimum number of chain guards required to sweep a simple polygon was given in [10], which actually contains a quadratic-time algorithm for reporting a sweep schedule of the two guards, if it exists.

Our objective in this paper is to find an optimum sweep, if a sweep exists, such that the sum of the distances travelled by the two guards in the sweep is minimized. This is the first work on the metric information for the two-guard and related problems, although it makes much use of some previously known techniques.[1] Apart from the theoretical interest, the motivation for studying this problem arises from the fact that the cost or energy required by a mobile robot (guard) is an increasing function of the distance it travels.

In Section 2 of this paper, we first give basic definitions [5,6]. In Sections 3 and 4, we show that any optimum sweep of the given polygon with two guards can be decomposed into at most $O(n)$ basic motions of the two guards, which are called the *canonical sweeps*. The canonical sweeps are the locally minimal motions of the two guards, which are defined between two rays (inside P) emanating from the reflex vertices of P. Next, we introduce a data structure that records the canonical sweeps and a transition relation among them (Section 4). By applying Dijkstra's algorithm to the obtained diagram, we obtain an $O(n^2)$ time and $O(n)$ space solution. Concluding remarks are given in Section 5.

2. Preliminaries

Let P denote a simple polygon of n vertices, and ∂P the boundary of P. Two points $p, q \in P$ are *visible* from each other if the line segment connecting them, denoted by $\overline{pq}$, does not intersect the exterior of P.

Let g_1, g_2 be two point guards on ∂P. Let $g_1(t)$ and $g_2(t)$ denote the positions of g_1 and g_2 on ∂P at time t; we require that $g_1(t)$ and $g_2(t): [0, \infty) \to \partial P$ be two continuous functions. A point $x \in P$ is said to be *detected* at t if x is contained in the line segment $\overline{g_1(t)g_2(t)}$. A *configuration* of g_1 and g_2 at time t is a pair of the points $g_1(t)$ and $g_2(t)$ such that the segment $\overline{g_1(t)g_2(t)}$ lies inside P. Assume that the initial positions of two guards are located at a vertex or on an edge of P. The configuration of g_1 and g_2 at a time then divides P into a *cleared* region that does not contain the target, and a *contaminated* region that may contain the target.

As in [6], a *sweep instruction* can be given by a pair of functions $g_1(t)$ and $g_2(t)$ such that $g_1(t)$ and $g_2(t)$ specify two edges of P along which g_1 and g_2 move respectively. More specifically, the following types of sweep instructions can be specified: The guards g_1 and g_2 move along the edges or some portions of them such that during the movement, (i) no intersections occur among all line segments $\overline{g_1(t)g_2(t)}$ or (ii) any two segments $\overline{g_1(t)g_2(t)}$ intersect [6].

Denote by $P(t)$ the fraction of the area of P that is cleared at time t. Initially, $P(0) = 0$. We say a *sweep schedule* exists for P, or equivalently, P allows a *sweep* if $P(t) = 1$ for some $t > 0$. The *complexity* of a sweep schedule is the total number of sweep instructions it consists of. For the sake of our problem (i.e., minimizing the sum of the travelled distances), we assume that the two guards start and end at a polygon vertex in a sweep schedule.

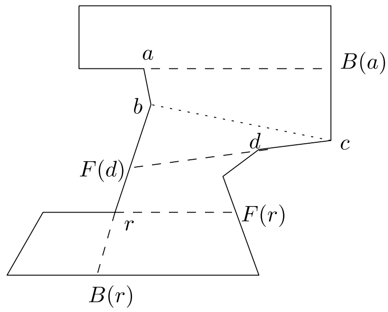

Fig. 1. Illustration for ray shots and canonical sweeps.

For a vertex v of P, denote by $Succ(v)$ and $Pred(v)$ the vertices immediately succeeding and preceding v *clockwise*, respectively. A polygon vertex is *reflex* if its interior angle is strictly greater than 180°. The *backward ray shot* from a reflex vertex r, denoted by $B(r)$, is defined as the first point of P hit by a "bullet" shot at r in the direction from $Succ(r)$ to r, and the *forward ray shot* $F(r)$ is the first point hit by the bullet shot at r in the direction from $Pred(r)$ to r. See Fig. 1. The vertex r is referred to as the *origin* of the shots $B(r)$ and $F(r)$, and $\overline{rB(r)}$ or $\overline{rF(r)}$ is referred to as a *ray-shooting segment*.

A sweep of the given polygon consists of a sequence of two basic motions of the guards, in which only instructions (i), or only instructions (ii) are used [5,6]. Since we are looking for an optimum sweep that minimizes the sum of the travelled distances, it is necessary and sufficient for either basic motion to start and end at a ray-shooting segment [6]. In this paper, we say a sweep is *straight* (resp. *counter*) if it moves the guards from a ray-shooting segment to another and only instructions (i) (resp. instructions (ii)) are used. Clearly, the total number of all possible straight and counter sweeps is bounded by $O(n^2)$.

Lemma 1. *(See [6,11].) A sweep of the given polygon with two guards can be decomposed into a sequence of straight and counter sweeps.*

A straight sweep from $\overline{ab}$ to $\overline{cd}$ is said to be *canonical* if its sweep schedule cannot be decomposed into the two, say, one from $\overline{ab}$ to $\overline{xy}$ and the other from $\overline{xy}$ to $\overline{cd}$, where $\overline{xy}$ is another ray-shooting segment. In Fig. 1, the straight sweep from $\overline{aB(a)}$ to $\overline{dF(d)}$ is canonical, but the sweep from $\overline{aB(a)}$ to $\overline{rF(r)}$ is not. The canonical sweep from $\overline{aB(a)}$ to $\overline{dF(d)}$ consists of two sweep instructions, which move the line segment connecting the guards from $\overline{aB(a)}$ to $\overline{bc}$ and then from $\overline{bc}$ to $\overline{F(d)d}$. Analogously, a counter sweep is *canonical* if its sweep schedule cannot be decomposed into two subschedules.

For the remainder of this paper, assume that the given polygon can be swept with two guards. Note that for two given ray-shooting segments, whether a straight or counter

[1] A preliminary version of this paper was presented at *FAW*'2010.