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Modular proof systems for partial functions with Evans equality

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Abstract

The paper presents a modular superposition calculus for the combination of first-order theories involving both total and partial functions. The modularity of the calculus is a consequence of the fact that all the inferences are pure—only involving clauses over the alphabet of either one, but not both, of the theories—when refuting goals represented by sets of pure formulae. The calculus is shown to be complete provided that functions that are not in the intersection of the component signatures are declared as partial. This result also means that if the unsatisfiability of a goal modulo the combined theory does not depend on the totality of the functions in the extensions, the inconsistency will be effectively found. Moreover, we consider a constraint superposition calculus for the case of hierarchical theories and show that it has a related modularity property. Finally, we identify cases where the partial models can always be made total so that modular superposition is also complete with respect to the standard (total function) semantics of the theories.
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1. Introduction

This paper aims at providing new modularity results for refutational theorem proving in first-order logic with equality. In Nelson–Oppen-style combinations of two first-order theories $\mathcal{T}_1$ and $\mathcal{T}_2$

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over signatures Σ_1 and Σ_2, inferences are *pure* in that all premises of an inference are clauses over only one of the signatures Σ_i where i depends on the inference. Therefore, no mixed formulae are ever generated when refuting goals represented by sets of pure formulae. What needs to be passed between the two theory modules are only universal formulae[1] over the intersection $\Sigma_1 \cap \Sigma_2$ of the two signatures. For stably infinite theories where, in addition, $\Sigma_1 \cap \Sigma_2$ consists of constants only, pure inference systems exist. This is one of the main consequences of Nelson and Oppen's results [23] (also see, e.g., Tinelli and Harandi [27] for additional clarification). The results have recently been extended to some situations when the theories $\mathcal{T}_1$ and $\mathcal{T}_2$ share also non-constant function symbols. Ghilardi [14] extended the completeness results for modular inference systems to a more general case of "compatibility" between the component theories $\mathcal{T}_i$. Future work might aim at weakening these compatibility requirements even further. In [26], Tinelli shows that similar modularity results are achieved if the theories share *all* their function symbols.

In this paper, we take a different point of departure. We will consider arbitrary theory modules $\mathcal{T}_1$ and $\mathcal{T}_2$ and investigate what one loses in terms of completeness when superposition inferences are restricted to be pure. Superposition is refutationally complete for equational first-order logic, and by choosing term orderings appropriately (terms over $\Sigma_1 \cap \Sigma_2$ should be minimal in the term ordering), many, but not all, cases of impure inferences can be avoided. Impure inferences arise when one of the extensions $\Sigma_1 \setminus \Sigma_2$ or $\Sigma_2 \setminus \Sigma_1$ has additional non-constant function symbols. It is known that in such cases interpolants of implications of the form $\phi_1 \supset \phi_2$, with ϕ_i a Σ_i-formula, normally contain existential quantification. That means, that refutationally complete clausal theorem provers where existential quantifiers are skolemized need to pass clauses from $\mathcal{T}_1$ to $\mathcal{T}_2$ [from $\mathcal{T}_2$ to $\mathcal{T}_1$] containing function symbols not in Σ_2 [Σ_1]. In other words, inference systems are necessarily either incomplete or impure.

One of the main results of the paper is that if the extensions only introduce additional relations and partial functions,[2] a particular calculus of superposition for partial functions to be developed in this paper becomes a complete and modular proof system where inferences are pure. This result can be applied to problems where partial functions arise naturally. Alternatively we may think of this result as indicating what we lose if superposition is restricted to pure inferences. If a proof cannot be found in the pure system, a partial algebra model exists for the goal to be refuted. Conversely, if the inconsistency of a goal does not depend on the totality of the functions in the extensions, we will be able to find the inconsistency with the modular partial superposition calculus. There are interesting cases of problem classes where partial models can always be totalized and where the modular system is therefore in fact complete (cf. Section 5).

[1] For Nelson–Oppen-style combination of theories, one even restricts the information exchange between theories to ground clauses over the intersection signature.

[2] A non-equational literal $p(t_1,\ldots,t_n)$ or $\neg\, p(t_1,\ldots,t_n)$, where p is a relation symbol, can be encoded as an equational literal $f_p(t_1,\ldots,t_n) \approx true_p$ or $\neg\, f_p(t_1,\ldots,t_n) \approx true_p$, where f_p is a function and $true_p$ a total constant. Thus we will in the sequel not mention relations anymore.

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