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Flip-pushdown automata with k pushdown reversals and E0L systems are incomparable

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ABSTRACT

We prove that any propagating E0L system cannot generate the language $\{w\#w \mid w \in \{0,1\}^*\}$. This result, together with some known ones, enables us to conclude that the flip-pushdown automata with k pushdown reversals, i.e., the pushdown automata with the ability to flip the pushdown, and E0L systems are incomparable. This result solves an open problem stated by Holzer and Kutrib in 2003.

1. Introduction

A flip-pushdown automaton, introduced by Sarkar [7], is an ordinary one-way pushdown automaton with the ability to flip its pushdown during the computation. It is known that the flip-pushdown automata without any limit on the number of flips are equally powerful to Turing machines [7].

Holzer and Kutrib have shown in [3,4] that $k+1$ pushdown reversals are more powerful than k for deterministic and nondeterministic flip-pushdown automata, and, nondeterminism is more powerful than determinism for flip-pushdown automata with constant number of flips. Moreover, they considered some closure properties and computational problems of these language families. However, they left several problems considered in [3] open. One of the open problems from [3] is the following: What is the relationship between E0L (or ET0L) languages and the languages accepted by flip-pushdown automata with constant number of flips? We give a definition of E0L systems below. For more information on L systems, see [6]. Although they have proved that the E0L language $\{a^n b^n c^n \mid n \geqslant 1\}$ cannot be recognized by such automata [3], they left open the second part of the problem.

In this paper we complete the solution of the problem mentioned above by showing that the language $\{w\#w \mid w \in \{0,1\}^*\}$ cannot be derived by any (propagating) E0L system. On the other hand, this language can be accepted by a pushdown automaton with one flip. To show that $\{w\#w \mid w \in \{0,1\}^*\}$ is not an E0L language, we use a proof technique that is quite different from techniques based on combinatorial properties of languages [2,6].

Note that, in [1,2,5,6], one can find several quite simple languages that are known not to be E0L languages, but it is not clear whether any of them is suitable for our purposes, i.e., acceptable by a flip-pushdown automaton with a constant number of flips.

2. Definitions

By $|M|$ we denote the cardinality of a set M. By $|x|$ we denote the length of a word x, and by λ we denote the empty word.

Definition 1. An *E0L system* is a quadruple $G = (\Sigma, P, \omega, \Delta)$, where Σ is a nonempty finite alphabet, $\omega \in \Sigma^*$, P is a finite set of productions of the form $\alpha \to \beta$, $\alpha \in \Sigma$, $\beta \in \Sigma^*$,

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and $\Delta \subseteq \Sigma$. If $\beta \neq \lambda$ for each production, then G is called *propagating*.

Definition 2. Let G be an E0L system $G = (\Sigma, P, \omega, \Delta)$. A *derivation* D in G is a triple (Θ, ν, p), where Θ is a finite set of ordered pairs of nonnegative integers (the position in D), ν is a function from Θ into Σ ($\nu(i, j)$ is the value of D at position (i, j)), and p is a function from Θ into P ($p(i, j)$ is production of D at position $\nu(i, j)$). Furthermore, there exists a sequence of words $\alpha_0, \alpha_1, \dots, \alpha_t$ in Σ^* (called the trace of D) such that $t \geqslant 1$ and the following conditions hold:

(i) $\Theta = \{(i, j) \mid 0 \leqslant i < t \text{ and } 1 \leqslant j \leqslant |\alpha_i|\}$,
(ii) $\nu(i, j)$ is the j-th symbol in α_i,
(iii) for $0 \leqslant i < t$, $\alpha_{i+1} = \delta_1\delta_2 \dots \delta_{|\alpha_i|}$, where $p(i, j)$ is the production $\nu(i, j) \to \delta_j$ for $1 \leqslant j \leqslant |\alpha_i|$.

In such a case D is said to be a *derivation* of α_t from α_0, and t is called the *length* of the derivation D. This is denoted by $\alpha_0 \Rightarrow_G^t \alpha_t$. Formally, $\alpha \Rightarrow_G^0 \alpha$ for each $\alpha \in \Sigma^*$. We will omit the subscript G [the superscript t] if G is clear from the context [if $t = 1$].

We will denote the language *generated* by E0L system G by $L(G)$, where

$$L(G) = \left\{x \,\middle|\, x \in \Delta^*, \omega \Rightarrow_G^t x \text{ for some } t \geqslant 0\right\}.$$

For some $i \in \{0, \dots, t-1\}$, let $\alpha_i = \gamma_1\gamma_2 \dots \gamma_{|\alpha_i|}$ ($\gamma_j \in \Sigma$ for $1 \leqslant j \leqslant |\alpha_i|$), and let $\alpha_{i+1} = \delta_1\delta_2 \dots \delta_{|\alpha_i|}$ be as in (iii) above. If $1 \leqslant d \leqslant h \leqslant |\alpha_i|$, we will say that the word $\delta_d\delta_{d+1} \dots \delta_h$ with position $(i+1, |\delta_1\delta_2 \dots \delta_{d-1}| + 1)$ is derived *under D* in one step from the word $\gamma_d\gamma_{d+1} \dots \gamma_h$ with position (i, d).

Let $0 \leqslant j < m \leqslant t$ and let $\alpha_i = \alpha_i^I\alpha_i^{II}\alpha_i^{III}$ for some α_i^I, $\alpha_i^{III} \in \Sigma^*$, $\alpha_i^{II} \in \Sigma^+$ for each $i \in \{j, \dots, m-1\}$. If the word α_{i+1}^{II} with position $(i+1, |\alpha_{i+1}^I| + 1)$ is derived under D in one step from the word α_i^{II} with position $(i, |\alpha_i^I| + 1)$ for each $i \in \{j, \dots, m-1\}$, then we will say that the word α_m^{II} with position $(m, |\alpha_m^I| + 1)$ is derived under D in $m - j$ steps from the word α_j^{II} with position $(j, |\alpha_j^I| + 1)$. If the positions will be clear from the context, we will omit that information.

3. Results

In this section we will prove the main result of this paper: E0L systems and languages accepted by flip-pushdown automata with a constant number of flips are incomparable. To do so, we will use the following theorem.

Theorem 1. *(See [3].) Any flip-pushdown automaton with a constant number of flips cannot accept the language* $L_1 = \{a^n b^n c^n \mid n \geqslant 1\}$.

On the other hand, L_1 is an E0L language. Consider an E0L system $G = (\Sigma, P, \omega, \Delta)$, where $\Sigma = \{a, b, c, A, B, C, A', B', C', F\}$, $\omega = ABC$, and $\Delta = \{a, b, c\}$. The set of productions P looks as follows:

$$\begin{array}{lll} A \to a \mid AA', & A' \to a \mid A', & a \to F, \\ B \to b \mid BB', & B' \to b \mid B', & b \to F, \\ C \to c \mid CC', & C' \to c \mid C', & c \to F, \\ & F \to F. & \end{array}$$

It is not hard to see that G generates L_1.

Consider the language $L_2 = \{w\#w \mid w \in \{0, 1\}^*\}$. The construction of a flip-pushdown automaton with one flip accepting L_2 is straightforward. Therefore, proving that L_2 is not an E0L language yields the incomparability result mentioned above. We will show that any propagating E0L system cannot generate L_2. Using the following theorem, which is a reformulation of Theorem 2.1 from [6], then directly implies that L_2 cannot be generated by any E0L system.

Theorem 2. *There is an algorithm that given any E0L system generating a language without the empty word produces a propagating E0L system generating the same language.*

To start with, we prove the following lemma and its corollary.

Lemma 1. *Consider a propagating E0L system* $G = (\Sigma, P, \omega, \Delta)$. *Let* $\alpha \Rightarrow^s \beta$ *for some* $\alpha, \beta \in \Sigma$ *and some* $s > (|\Sigma|!) \cdot |\Sigma|^2$. *Then* $\alpha \Rightarrow^{s'} \beta$ *for* $s' = s - |\Sigma|!$.

Proof. Since G is a propagating E0L system, there is a derivation D with a trace $\alpha = \alpha_0, \alpha_1, \dots, \alpha_s = \beta$, where $\alpha_i \in \Sigma$ for all $i \in \{0, \dots, s\}$. For every $j \in \{0, 1, 2, \dots, (|\Sigma|!) \cdot |\Sigma| - 1\}$, the sequence $\alpha_{t_j}, \alpha_{t_j+1}, \alpha_{t_j+2}, \dots, \alpha_{t_j+|\Sigma|}$, where $t_j = j|\Sigma|$, must contain two elements α_{l_j}, α_{m_j} such that $t_j \leqslant l_j < m_j \leqslant t_{j+|\Sigma|}$, and $\alpha_{l_j} = \alpha_{m_j}$. For $i \in \{1, 2, \dots, |\Sigma|\}$, let $B_i = \{j \mid 0 \leqslant j \leqslant (|\Sigma|!) \cdot |\Sigma| - 1, m_j - l_j = i\}$. Since $1 \leqslant m_j - l_j \leqslant |\Sigma|$ for each j, there is a set B_r with $|B_r| \geqslant (|\Sigma|!) \cdot |\Sigma| / |\Sigma| = |\Sigma|!$. Since $\alpha_{l_j} = \alpha_{m_j}$, modifying D in such a way that the segment $\alpha_{l_j+1}, \alpha_{l_j+2}, \dots, \alpha_{m_j}$ with $j \in B_r$ is deleted from the trace, we obtain a derivation of β from α of length $s - r$. Thus, by deleting $|\Sigma|!/r$ such segments, we obtain a derivation of β from α of length $s - |\Sigma|!$. □

Corollary 1. *Consider a propagating E0L system* $G = (\Sigma, P, \omega, \Delta)$. *There exists a constant* $c > 0$ *such that if* $\omega \Rightarrow^h x$ *for some* $x \in \Sigma^+$ *and for some* $h \geqslant 0$, *then* $\omega \Rightarrow^g x$ *for some* $g \leqslant c|x|$.

Proof. Let D be the shortest derivation of x from ω. Denote by d its length. Since G is a propagating E0L system, there are words $\psi_1, \psi_1', \psi_2, \psi_2', \dots, \psi_t, \psi_t'$ such that

$$\begin{aligned} \omega = \psi_1 \Rightarrow^{l_1} \psi_1' \Rightarrow \psi_2 \Rightarrow^{l_2} \psi_2' \Rightarrow \psi_3 \Rightarrow^{l_3} \psi_3' \\ \Rightarrow \dots \Rightarrow \psi_t \Rightarrow^{l_t} \psi_t' = x, \end{aligned}$$

where $t \geqslant 1$, $|\psi_i| = |\psi_i'|$, $l_i \geqslant 0$ for all $i \in \{1, \dots, t\}$, $|\psi_i'| < |\psi_{i+1}|$ for all $i \in \{1, \dots, t-1\}$, and $d = t - 1 + \sum_{i=1}^t l_i$. Suppose to the contrary that $d > c|x|$ for $c = (|\Sigma|!) \cdot |\Sigma|^2 + 2$. Thus, $l_j \geqslant c - 1$ for some j, because $t \leqslant |x|$. Let $\psi_j = \delta_1\delta_2 \dots \delta_m$ and $\psi_j' = \delta_1'\delta_2' \dots \delta_m'$, where $m = |\psi_j| = |\psi_j'|$ and $\delta_i, \delta_i' \in \Sigma$ for all $i \in \{1, \dots, m\}$. Since G is propagating,