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Routing and wavelength assignment for 3-ary n-cube communication patterns in linear array optical networks for n communication rounds

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ABSTRACT

k-ary n-cubes are a class of communication patterns that are employed by a number of typical parallel algorithms. This paper addresses the implementation of parallel algorithms with bidirectional 3-ary n-cube communication patterns on a bidirectional linear array WDM optical networks when the information is transmitted one dimension after another. By giving an embedding scheme ϕ, we prove the optimal number of wavelengths under ϕ and design a routing and wavelength assignment strategy of it.

1. Introduction

In a typical WDM optical network, every fiber link can support a certain number of wavelengths, and each wavelength can carry a separate stream of data. To efficiently utilize the bandwidth resources and to eliminate the high cost and bottleneck caused by opoelectrical conversion and processing at intermediate nodes, end-to-end lightpaths are usually set up between each pair of source–destination nodes. A connection or lightpath in a WDM network is an ordered pair of nodes $\langle x, y\rangle$ corresponding to transmission of a packet from source x to destination y.

The central task for WDM optical networks is to select a suitable path and wavelength for each connection of a given communication pattern so that the number of wavelengths used is minimized, with the following constrains: (1) Wavelength-continuity constraint. (2) Distinct wavelength constraint. There are some results about routing and wavelength assignments in optical networks when only one communication round is used in [2,3,9,10]. But in reality, the number of wavelengths feasible is limited. So when the connections are large enough, the number of wavelengths required will outnumber that of the network can afford. In [4–6,8] the authors considered wavelength assignment for parallel FFT communication pattern on a class of regular optical networks by giving some embedding schemes.

This paper addresses the routing and wavelength assignment for bidirectional Q_n^3 communication patterns in linear array WDM optical networks when n communication rounds are used. This paper confines that the information is transmitted one dimension after another. By giving an embedding scheme ϕ and a routing and wavelength assignment strategy, we prove that the number of wavelengths under ϕ is $4\times 3^{n-2}+2$.

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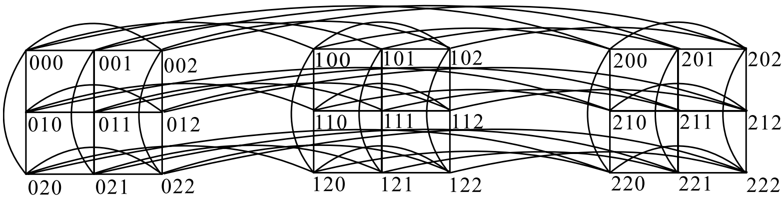

Fig. 1. 3-ary 3-cube.

The rest of this paper is organized as follows. In Section 2, some preliminaries are introduced. In Section 3, we design embedding scheme ϕ and provide some properties. In Section 4, the congestion about dimensions under ϕ is obtained. In Section 5, the optimal number of wavelengths under ϕ is achieved when the packets are transmitted one dimension after another and a routing and wavelength assignment strategy is designed. Finally, we conclude this paper in Section 6.

2. Preliminary

Definition 1. (See [7].) The k-ary n-cube Q_n^k ($k \geqslant 2$ and $n \geqslant 1$) has $N = k^n$ vertices, each of the form $x = (x_{n-1}, x_{n-2}, \ldots, x_0)$, where $0 \leqslant x_i \leqslant k-1$ for every $0 \leqslant i \leqslant n-1$. Two vertices $x = (x_{n-1}, x_{n-2}, \ldots, x_0)$ and $y = (y_{n-1}, y_{n-2}, \ldots, y_0)$ in Q_n^k are adjacent if and only if there exists an integer j, $0 \leqslant j \leqslant n-1$, such that $x_j = y_j \pm 1 \pmod k$ and $x_i = y_i$ for $i \in \{0, 1, 2, \ldots, n-1\} - \{j\}$.

Fig. 1 depicts a 3-ary 3-cube.

Definition 2. (See [9].) The natural numbering of Q_n^k assigns to each vertex $x = (x_{n-1}, x_{n-2}, \ldots, x_0)$ the number $1 + \sum_{i=0}^{n-1} x_i k^i$.

Denote by $DIM_{n,i}$, where $0 \leqslant i \leqslant n-1$, the set of all the corresponding dimension i edge. That is:

$$E(Q_n^3) = \bigcup_{i=0}^{n-1} DIM_{n,i}$$

$$DIM_{n,i} = \big\{\big(j, j+3^i\big), \big(j, j+2\times 3^i\big), \big(j+3^i, j+2\times 3^i\big) \bigm| j \bmod 3^{i+1} < 3^i\big\}$$

The bidirectional 3-ary n-cube, $Q_n^{3,b}$, is a bidirectional orientation of Q_n^3. Let $DIM_{n,i}^b$ denote the set of all directed edges of dimension i in $Q_n^{3,b}$. Then

$$E\big(Q_n^{3,b}\big) = \bigcup_{i=0}^{n-1} DIM_{n,i}^b$$

$$DIM_{n,i}^b = \big\{\big\langle j, j+3^i\big\rangle, \big\langle j+3^i, j\big\rangle, \big\langle j, j+2\times 3^i\big\rangle, \big\langle j+2\times 3^i, j\big\rangle, \big\langle j+3^i, j+2\times 3^i\big\rangle, \big\langle j+2\times 3^i, j+3^i\big\rangle \bigm| j \bmod 3^{i+1} < 3^i\big\}$$

Let $G = (V(G), E(G))$ be the guest graph, $H = (V(H), E(H))$ the host graph, where $|V(G)| = |V(H)|$. An embedding scheme of G in H is an ordered pair $\phi = (\psi, \varphi)$, where ψ is a bijection from $V(G)$ to $V(H)$, φ is a mapping from $E(G)$ to a set of paths in H such that $\varphi(u, v)$ is a path from $\psi(u)$ to $\psi(v)$ if $(u, v) \in E(G)$.

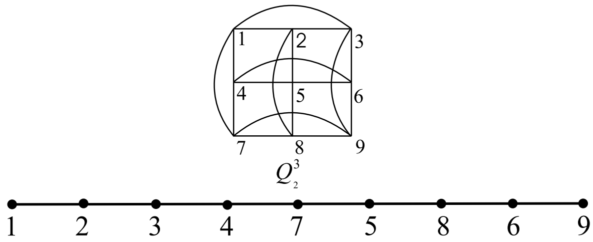

Fig. 2. Embedding Q_2^3 into L_2 under ϕ.

Define the congestion of an edge e on dimension i of Q_n^3 under embedding scheme ϕ of G in H as

$$D_i - Cong\big(Q_n^3, G, \phi, e\big) = \big|\big\{f \in DIM_{n,i} \bigm| e \in \varphi(f)\big\}\big|$$

the congestion on dimension i of Q_n^3 in G under ϕ as

$$D_i - Cong\big(Q_n^3, G, \phi\big) = \max_{e \in DIM_{n,i}} \big\{D_i - Cong\big(Q_n^3; G; \phi; e\big)\big\}$$

the congestion about dimensions of Q_n^3 in G under ϕ as

$$D - Cong\big(Q_n^3; G; \phi\big) = \max_i \big\{D_i - Cong\big(Q_n^3; G; \phi\big)\big\}$$

Let $\lambda_{n,\phi}(Q_n^3; G)$ denote the number of wavelengths required when the information is transmitted one dimension after another for realizing communication pattern of Q_n^3 in optical network G by embedding scheme ϕ. Let L_n denote a linear array of 3^n vertices. All the edges of L_n are labeled as $e_1, e_2, \ldots, e_{3^n-1}$ from left to right.

3. Embedding scheme ϕ of Q_n^3 in L_n

First, we give an embedding scheme ϕ of Q_n^3 in L_n as follows:

(1) Embed the vertex whose natural numbering is 1 into the first vertex of L_n, let $i = 1$;

(2) Embed the vertices which are not embedded into L_n before such that all the vertices are adjacent to i, and the natural numberings of them increase one by one. Denote by S_i the set of all these vertices;

(3) If $i < 3^{n-1}$, then let $i = i + 1$, goto (2);

It is obvious that all the vertices of Q_n^3 have been embedded into L_n. In the following, we consider the embedding scheme ϕ as given in the above (embedding Q_2^3 into L_2 is shown in Fig. 2). Denote $\kappa_i^m = \max_{e_j=(u,v), u,v \in S_i}\{D_m - Cong(Q_n^3; L_n; \phi; e_j)\}$, it is obvious that $\kappa_{3^{n-1}}^{n-1} = 2$.