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Fault tolerance analysis of mesh networks with uniform versus nonuniform node failure probability ☆

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ABSTRACT

Mesh networks have been applied to build large scale multicomputer systems and Network-on-Chips (NoCs). Mesh networks perform poorly in tolerating faults in the view of worst-case analysis, so it is practically important for multicomputer systems and NoCs manufactures to determine the lower bound for the mesh network connectivity probability when the node failure probability and the network size are given. In this paper, we study the topic based on *k*-submesh model under two fault models: Each node has uniform or nonuniform failure probability. We develop a novel technique to formally derive lower bound on the connectivity probability for mesh networks. Our study shows that mesh networks of practical size can tolerate a large number of faulty nodes and maintain higher connectivity probability, thus are reliable and trustworthy enough for multicomputer systems and NoCs. For example, suppose we are building a mesh network of 40 000 nodes (e.g., $M_{200\times 200}$) and require a network connectivity probability 99%, we only need to bound the uniform node failure probability by 0.25%. On the other hand, for the same size network $M_{200\times 200}$, the mesh network connectivity probability can maintain 95.88% even the network runs one million seconds uninterruptedly under exponential distribution node failure probability with failure rate 10^{-9} level.

1. Introduction

In recently years, system trustworthiness has been presented and studied extensively. In fact, system trustworthiness is a combination of several attributes: reliability, safety, security, availability, performance, fault tolerance, and privacy etc. Some of these attributes can be directly measured, some cannot. For example, performance, fault tolerance and availability can be numerically measured, but safety and security cannot.

In this paper, we will concentrate on the fault tolerance of mesh networks. The advantages of mesh networks include their simplicity, regularity and good scalability, so mesh networks are among the most important and attractive interconnection network topologies for large multicomputer systems. A number of large research and commercial multicomputer systems have been built based on mesh topologies, including Infiniband, Myrinet, Illiac IV, Intel Paragon, Stanford DASH, Goodyear MPP, Tera Computer System, Intel Touchstone Delta, MIT Alewife, Cray T3D, Blue Gene Supercomputer, and MasPar series [1–4]. In particular, current VLSI technology allows to fabricate large scale Network-on-Chips (NoCs) devices, integrating thousands of cores into a single chip based on mesh interconnection networks, such as Intel Corp's TeraFLOPS and Tilera Corp's TILE64 [5,6]. Due to inevitable node failures, fault tolerance of mesh networks is an unavoidable topic in system trustworthiness attributes to build reliable and trustworthy large scale multicomputer systems and NoCs. For example, multicomputer systems and NoCs manufactures must

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determine the lower bound for the mesh network connectivity probability when the node failure probability and the network size are given. So, the problems of fault tolerance or developing efficient and reliable routing algorithms in a mesh network for multicomputers and NoCs with faults has been extensively studied in the last two decades [7–20].

The paper is organized as follows: Section 2 reviews related work. In Section 3, we give some concepts about "k-submesh". Based on the k-submesh connectivity, we derive the connectivity probability for mesh networks in terms of node failure probability in Section 4. In Section 5, we study the probability of mesh network connectivity based on uniform and nonuniform failure probability. Conclusions are given in Section 6.

2. Related work

In the aspect of multicomputer systems, R. Boppana and S. Chalasani proposed the most classical fault block model and fault-tolerant routing algorithm with two virtual channels [7]. Subsequently, the results have further been extended, fault block model maybe solid faulty shape, such as "+", "L" and "T", but the new algorithm must use four virtual channels without overlapping fault chain. C. Chen and G. Chiu improved the results, and their algorithm only uses three virtual channels with overlapping faults chain [8]. R.L. Hadas proposed a fault-tolerant model called "Origin" concept for mesh networks [9]. The algorithm can tolerate at least $(k-1)^{n-1}$ faulty nodes in n-dimensional mesh networks. G.J. Glass and L.M. Ni considered an adaptive fault-tolerant routing algorithm based on a "Turn" model [10]. The Turn model produces routing algorithms which are deadlock-free, adaptive, minimal or non-minimal, and livelock-free for direct networks no matter whether they involve virtual channels or not. In addition, the routing algorithms produced by the Turn model can deal with dynamic faulty nodes. Practical Deadlock-Free Fault-Tolerant Routing is proposed based on the Planar Network Fault Model [11]. In [12], the authors studied that how to directly embed a generalized Fibonacci cube (GFC) into a faulty hypercube, and proposed an $O(n)$ time sequential algorithm for embedding a maximum GFC into a faulty n-cube with no more than three faulty nodes when $n=4$ or $n \geqslant 6$. For a practical parallel system constructed with an interconnection network, the probability of more than three faulty processors at a time is very low. Hence, the result derived by the authors is very applicable in practice for a parallel system.

In the aspect of NoCs, the authors utilize the region concept to design fault-tolerant algorithms in [13,14]. In [15], the authors discussed the implementation of techniques for detection and recovery of faults in a NoC. In [16], a routing scheme called MinFT is proposed, which adapts as per-link failures while following the minimal path and reserving bandwidth as per QoS requirement in NoCs. The highlight of the scheme is the continuation of functioning of NoCs even in case of link failure as well as node failures for different types of traffic. In [17], the authors enhanced available message-based approach for NoCs architectures without using virtual channels. In [18], the authors proposed a novel routing control algorithm for non-VC router of irregular 2D-mesh NoCs. In [19], the authors proposed a distributed routing algorithm for NoCs, allowing a network to reconfigure around faulty components. Experimental results showed an average reliability of over 99.99% when 10% of the network links have failed under different networks sizes.

Due to some disadvantages of mesh topology, some assumptions or restrictions in proposed fault-tolerant model are adopted when researchers study the fault tolerance and the fault-tolerant routing algorithms of mesh networks in order to avoid faulty nodes, such as: (1) restrict the shape of faulty block: "+", "L" and "T"; (2) use virtual channels; (3) use other models and restrictions. In general, adding virtual channels to mesh is not free, which involves adding buffer space and complex control logic to the nodes, and the addition of extra logic circuits and buffer space makes nodes more liable to fail and less reliable. In addition, if the shape of faulty pattern is confined then many non-faulty nodes will be sacrificed.

In this paper, we will investigate the mesh network connectivity based on our proposed k-submesh model. The advantage of the k-submesh is that we can reduce the difficult task of determining the connectivity of the entire large network to the easier task of determining the connectivity of much smaller submeshes. Then, we calculate the probability of k-submesh connectivity for each k-submesh. Since each k-submesh is small, this can be done either by simpler combinatorial analysis or by exhaustive enumeration. From the probability of k-submesh connectivity, we will be able to derive lower bound for the probability for the connectivity of the entire mesh network.

3. On *k*-submesh connectivity

In this paper, $M_{m\times n}$ denote a mesh network of size $m\times n$ with mn nodes. Each node in $M_{m\times n}$ is given by a coordinate pair (x, y), where $1 \leqslant x \leqslant m$ and $1 \leqslant y \leqslant n$. Two nodes $v=(x, y)$ and $v'=(x', y')$ in $M_{m\times n}$ are *neighboring* if $|x-x'|+|y-y'|=1$, i.e., if v and v' are identical in one coordinate while the values for the other coordinate differ by 1. Then we introduce the concept of k-submesh connectivity in mesh network $M_{m\times n}$. Without loss of generality, we assume that the values of m and n are all divisible by k.

Definition 3.1. A *k-submesh* M_k (or sometimes more specifically $M_{a,b}^{(k)}$) in the mesh network $M_{m\times n}$ is determined by two integers a and b, $0 \leqslant a < m/k$ and $0 \leqslant b < n/k$, such that $M_{a,b}^{(k)}$ consists of the k^2 nodes (x, y) in $M_{m\times n}$, where $ak+1 \leqslant x \leqslant (a+1)k$ and $bk+1 \leqslant y \leqslant (b+1)k$.

Thus, a k-submesh in $M_{m\times n}$ is a $k\times k$ squared mesh, in which we can naturally define the four "sides" of the square. Two k-submeshes $M_{a,b}^{(k)}$ and $M_{a',b'}^{(k)}$ are *neighboring* if either $a=a'$ and $|b-b'|=1$, or $|a-a'|=1$ and $b=b'$. Fig. 1 illustrates the 3-submesh partition and neighboring 3-submeshes in the mesh network $M_{6\times 9}$.

Definition 3.2. A mesh network $M_{m\times n}$ is *k-submesh connected* if for each k-submesh M_k in $M_{m\times n}$, the non-faulty