Information Processing Letters 114 (2014) 229–233

Contents lists available at ScienceDirect

Information Processing Letters

www.elsevier.com/locate/ipl

On the topological complexity of ω-languages of non-deterministic Petri nets

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ARTICLE INFO

Article history:
Received 24 October 2012
Received in revised form 29 November 2013
Accepted 12 December 2013
Available online 28 December 2013
Communicated by A. Muscholl

Keywords:
Formal languages
Languages of infinite words
Petri nets
Büchi acceptance condition
Cantor space
Topological complexity
Borel hierarchy, complete sets

ABSTRACT

We show that there are $\mathbf{\Sigma}_3^0$-complete languages of infinite words accepted by non-deterministic Petri nets with Büchi acceptance condition, or equivalently by Büchi blind counter automata. This shows that ω-languages accepted by non-deterministic Petri nets are topologically more complex than those accepted by deterministic Petri nets.

1. Introduction

The languages of infinite words, also called ω-languages, accepted by finite automata were first studied by Büchi to prove the decidability of the monadic second order theory of one successor over the integers. Since then regular ω-languages have been much studied and used for specification and verification of non-terminating systems, see [21,20,15] for many results and references. The acceptance of infinite words by other finite machines, like pushdown automata, counter automata, Petri nets, Turing machines, ..., with various acceptance conditions, has also been considered, see [20,5,2,9].

Since the set Σ^ω of infinite words over a finite alphabet Σ is naturally equipped with the Cantor topology, a way to study the complexity of languages of infinite words accepted by finite machines is to study their topological complexity and firstly to locate them with regard to the Borel and the projective hierarchies [21,19,5,13,20,18,17].

Every ω-language accepted by a deterministic Büchi automaton is a $\mathbf{\Pi}_2^0$-set. On the other hand it follows from Mac Naughton's Theorem that an ω-language accepted by a non-deterministic Büchi (or Muller) automaton is also accepted by a deterministic Muller automaton, and thus is a boolean combination of ω-languages accepted by deterministic Büchi automata. Therefore every ω-language accepted by a non-deterministic Büchi (or Muller) automaton is a $\mathbf{\Delta}_3^0$-set. In a similar way, every ω-language accepted by a deterministic Muller Turing machine, and thus also by any Muller deterministic finite machine is a $\mathbf{\Delta}_3^0$-set, [5,20].

We consider here acceptance of infinite words by Petri nets or equivalently by (partially) blind counter automata. Petri nets are used for the description of distributed systems [6,16,11], and they may be defined as partially blind multicounter automata, as explained in [22,5,10]. In order to get a partially blind multicounter automaton which

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[1] This author has been supported by National Science Centre grant No. DEC-2012/05/N/ST6/03254.

0020-0190/$ – see front matter
http://dx.doi.org/10.1016/j.ipl.2013.12.007

accepts the same language as a given Petri net, one can distinguish between the places of a Petri net by dividing them into the bounded ones (the number of tokens in such a place at any time is uniformly bounded) and the unbounded ones. Then each unbounded place may be seen as a partially blind counter, and the tokens in the bounded places determine the state of the partially blind multicounter automaton. The transitions of the Petri net may then be seen as the finite control of the partially blind multicounter automaton and the labels of these transitions are then the input symbols.

The infinite behavior of Petri nets was first studied by Valk [22] and by Carstensen in the case of deterministic Petri nets [1].

On one side the topological complexity of ω-languages of deterministic Petri nets is completely determined. They are $\mathbf{\Delta}^0_3$-sets and their Wadge hierarchy, which is a great refinement of the Borel hierarchy, defined via reductions by continuous functions, has been determined in [7,3,4]; its length is the ordinal ω^{ω^2}.

On the other side, nothing was known about the topological complexity of ω-languages of non-deterministic Petri nets. We show that there exist $\mathbf{\Sigma}^0_3$-complete, hence non-$\mathbf{\Delta}^0_3$, ω-languages accepted by one-blind-counter Büchi automata. Notice that it was proved in [8] that ω-languages accepted by (non-blind) one-counter Büchi automata have the same topological complexity as ω-languages of Turing machines, but the non-blindness of the counter was essential in the proof since the ability to use the zero-test of the counter was important.

This provides the first result on the topological complexity of ω-languages of non-deterministic Petri nets and shows that there exist some ω-languages accepted by non-deterministic Petri nets, and even by one-blind-counter Büchi automata, which are topologically more complex than those accepted by deterministic Petri nets.

2. Basic notions

We assume the reader to be familiar with the theory of formal (ω)-languages, see [21,20].

When Σ is a countable alphabet, a *non-empty finite word* over Σ is any sequence $x = a_1 \ldots a_k$, where $a_i \in \Sigma$ for $i = 1, \ldots, k$, and k is an integer $\geqslant 1$. $\Sigma^\star$ is the *set of finite words* (including the empty word ε) over Σ.

The *first infinite ordinal* is ω. An *ω-word* over Σ is an ω-sequence $a_1 \ldots a_n \ldots$, where for all integers $i \geqslant 1$, $a_i \in \Sigma$. When σ is an ω-word over Σ, we write $\sigma = \sigma(1)\sigma(2)\ldots\sigma(n)\ldots$, where for all i, $\sigma(i) \in \Sigma$, and $\sigma[n] = \sigma(1)\sigma(2)\ldots\sigma(n)$.

The concatenation product of two finite words u and v is denoted $u \cdot v$ and sometimes just uv. This product is extended to the product of a finite word u and an ω-word v: the infinite word $u \cdot v$ is then the ω-word such that: $(u \cdot v)(k) = u(k)$ if $k \leqslant |u|$, and $(u \cdot v)(k) = v(k - |u|)$ if $k > |u|$.

The *set of ω-words* over the alphabet Σ is denoted by Σ^ω. An *ω-language* over an alphabet Σ is a subset of Σ^ω.

A blind multicounter automaton is a finite automaton equipped with a finite number (k) of blind (sometimes called partially blind, as in [10]) counters. The content of any such counter is a non-negative integer. A counter is said to be blind when the multicounter automaton cannot test whether the content of the counter is zero. This means that if a transition of the machine is enabled when the content of a counter is zero then the same transition is also enabled when the content of the same counter is a non-zero integer.

We now give the definition of a Büchi 1-blind-counter automaton. Notice that we consider here only real time automata, i.e., without ε-transitions.

Definition 2.1. A (real time) Büchi 1-blind-counter automaton is a 5-tuple $\mathcal{A} = (Q, \Sigma, \Delta, q_0, F)$, where Q is a finite set of states, Σ is a finite input alphabet, $q_0 \in Q$ is the initial state, the transition relation Δ is a subset of $Q \times \Sigma \times \{0,1\} \times Q \times \{0,1,-1\}$, and $F \subseteq Q$ is the set of accepting states.

If the automaton $\mathcal{A}$ is in state q, and $c \in \mathbb{N}$ is the content of the counter $\mathcal{C}$, then the configuration (or global state) of $\mathcal{A}$ is the pair (q, c).

Given any $a \in \Sigma$, any $q, q' \in Q$, and any $c \in \mathbb{N}$, if both $\Delta(q, a, i, q', j)$, and $(c \geqslant 1 \Rightarrow i = 1)$ and $(c = 0 \Rightarrow (i = 0$ and $j \in \{0,1\}))$ hold, then we write: $a : (q, c) \mapsto_{\mathcal{A}} (q', c + j)$.

Moreover the counter of $\mathcal{A}$ is blind, *i.e.*, if $\Delta(q, a, i, q', j)$ holds, and $i = 0$ then $\Delta(q, a, i', q', j)$ holds also for $i' = 1$.

Let $x = a_1 a_2 \ldots a_n \ldots$ be an ω-word over Σ. An ω-sequence of configurations $\rho = (q_i, c_i)_{i \geqslant 1}$ is called a run of $\mathcal{A}$ on x if and only if

- $(q_1, c_1) = (q_0, 0)$, and
- $a_i : (q_i, c_i) \mapsto_{\mathcal{A}} (q_{i+1}, c_{i+1})$ (for all $1 \leqslant i$).

We denote $In(\rho)$ the set of all the states visited infinitely often during the run ρ. The automaton $\mathcal{A}$ accepts x if there is an infinite run ρ of $\mathcal{A}$ on x such that $In(\rho) \cap F \neq \emptyset$.

The ω-language accepted by $\mathcal{A}$ is the set $L(\mathcal{A})$ of ω-words accepted by $\mathcal{A}$.

We assume the reader to be familiar with basic notions of topology which may be found in [14,13,12,20,15]. If X is a countable alphabet containing at least two letters, then the set X^ω of infinite words over X may be equipped with the product topology of the discrete topology on X. This topology is induced by a natural metric which is called the *prefix metric* and defined as follows. For $u, v \in X^\omega$ and $u \neq v$ let $\delta(u, v) = 2^{-l_{\mathrm{pref}(u,v)}}$ where $l_{\mathrm{pref}(u,v)}$ is the first integer n such that the $(n+1)$st letter of u is different from the $(n+1)$st letter of v.

If X is finite then X^ω is a Cantor space and if X is countably infinite then X^ω is homeomorphic to the Baire space ω^ω. The open sets of X^ω are the sets in the form $W \cdot X^\omega$, where $W \subseteq X^\star$.

The classes $\mathbf{\Sigma}^0_n$ and $\mathbf{\Pi}^0_n$ of the Borel Hierarchy on the topological space X^ω are defined as follows: $\mathbf{\Sigma}^0_1$ is the class of open sets of X^ω, $\mathbf{\Pi}^0_1$ is the class of closed sets (i.e. complements of open ones) of X^ω. And for any integer $n \geqslant 1$: $\mathbf{\Sigma}^0_{n+1}$ is the class of countable unions of $\mathbf{\Pi}^0_n$-subsets of X^ω, and $\mathbf{\Pi}^0_{n+1}$ is the class of countable intersections of $\mathbf{\Sigma}^0_n$-subsets of X^ω. The Borel Hierarchy is also defined