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Locally exchanged twisted cubes: Connectivity and super connectivity ☆

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ABSTRACT

Connectivity $\kappa(G)$ (resp., super connectivity $\kappa'(G)$) of a graph G is the minimum number of vertices whose removal leaves the remaining graph disconnected or trivial (resp., the remaining graph disconnected and without isolated vertex). These two parameters are important for interconnection networks and can be used to measure reliability in such networks. In this paper, a new interconnection network called locally exchanged twisted cube (LETQ for short), denoted $LeTQ(s,t)$, is proposed. We obtain some basic properties of LETQ including isomorphism, decomposition, Hamiltonicity and connectivity. In particular, we determine $\kappa(LeTQ(s,t)) = \min\{s+1, t+1\}$ and $\kappa'(LeTQ(s,t)) = \min\{2s, 2t\}$ for $s, t \geqslant 1$.

1. Introduction

Interconnection networks play an important role in parallel computing systems, and such a network can be modeled by a graph $G = (V, E)$, where V (or $V(G)$) represents the set of processors and E (or $E(G)$) is the set of communication links between processors. It is well-known that hypercubes are the most popular and efficient interconnection networks due to their rich topological properties, such as regularity, symmetry, small diameter, strong connectivity, recursive structure, flexible partition, and relatively low link complexity [21]. To overcome some shortcomings of a normal hypercube, variations of hypercube architecture have been proposed for achieving the improvement on their efficiency. For more previous results on variations of hypercubes, the reader can refer to [2,6].

As a variant of hypercube, the family of locally twisted cubes was first introduced by Yang et al. [26]. It was shown that the diameter of an n-dimensional locally twisted cube, denoted as LTQ_n, is only about half of that of the corresponding hypercube. Another advantage is that the rule of adjacency for vertices in LTQ_n is simpler than other variations. In particular, any two adjacent vertices in LTQ_n differ only in at most two successive bits. More attractive properties and application support merits of LTQ_n can be found in the literature, e.g., studies on diagnosability [25], mesh embedding [3], Hamiltonicity [5,17,18,24,27], and independent spanning trees [4,11,12].

Recently, exchanged hypercube $EH(s,t)$, proposed by Loh et al. [14] is a new interconnection network obtained by systematically removing links from a hypercube. The structure of exchanged hypercubes not only kept numerous desirable properties of the normal hypercubes, but also reduced the interconnection complexity. In particular, re-

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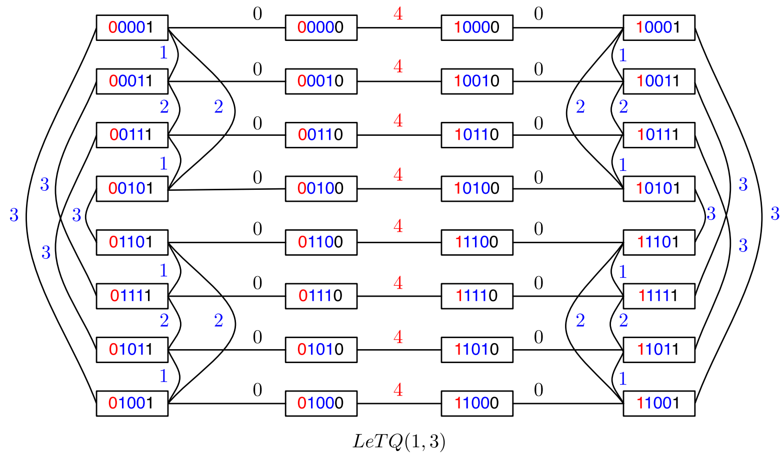

Fig. 1. Locally exchanged twisted cube $LeTQ(1,3)$.

searches of exchanged hypercubes have been devoted on the topics including domination [7,8], connectivity [10,15,19], cycle embedding [16], edge congestion [23], wide and fault diameter [22], and others [1,13]. In addition, a variant of exchanged hypercube called exchanged crossed cubes has been studied recently in [9,20].

In this paper, we combine the notion of locally twisted cubes and exchanged hypercubes to introduce a novel network topology called locally exchanged twisted cube (LETQ for short). We obtain some basic properties of LETQ including isomorphism, Hamiltonicity, decomposition and connectivity. Especially, connectivity is an important parameter for interconnection networks and it can be used to measure reliability in such networks. Hence, we dedicate ourselves to study the connectivity and a restricted case called super connectivity of LETQ.

The rest of this paper is organized as follows. Section 2 gives the definition of LETQ and provides some basic properties of LETQ. Section 3 proves the main results of finding the connectivity and super connectivity in LETQ. Finally, concluding remarks are given in Section 4.

2. Locally exchanged twisted cubes

For convenience, vertices in a hypercube or its variants are usually encoded by using binary string representation. Also, we use the notation G^x to denote the labeled graph obtained from a graph G by prefixing the binary string of every vertex with x. Let $\oplus$ denote the modulo 2 addition. The *n-dimensional locally twisted cube* LTQ_n is defined as follows:

(1) LTQ_2 is a graph consisting of four vertices with labels 00, 01, 10, 11 together with four edges (00, 01), (00, 10), (01, 11), and (10, 11).
(2) For $n \geqslant 3$, LTQ_n is constructed from two copies of LTQ_{n-1} (i.e., LTQ^0_{n-1} and LTQ^1_{n-1}) by the following rule: each vertex $x = 0x_{n-2}x_{n-3}\cdots x_0$ in LTQ^0_{n-1} is connected with the vertex $1(x_{n-2}\oplus x_0)x_{n-3}\cdots x_0$ in LTQ^1_{n-1} by an edge.

Inspired by the idea of exchanged hypercube, we pose the following definition.

Definition 1. A *locally exchanged twisted cube* is an undirected graph $LeTQ(s,t) = G(V,E)$, where $s,t \geqslant 1$, $V = \{x = x_{t+s}\cdots x_{t+1}x_t\cdots x_1x_0 \colon x_i \in \{0,1\}$ for $0 \leqslant i \leqslant t+s\}$ is the vertex set, and E is the edge set composed of the following three types of disjoint sets E_1, E_2 and E_3:

$$E_1 = \{(x,y) \in V \times V \colon x \oplus y = 2^0\},$$

$$\begin{aligned}E_2 = \{&(x,y) \in V \times V \colon x_0 = y_0 = 1, x_1 = y_1 = 0\\ &\text{and } x \oplus y = 2^k \text{ for } k \in [3,t]\} \cup\\ &\{(x,y) \in V \times V \colon x_0 = y_0 = x_1 = y_1 = 1\\ &\text{and } x \oplus y = 2^k + 2^{k-1} \text{ for } k \in [3,t]\} \cup\\ &\{(x,y) \in V \times V \colon x_0 = y_0 = 1 \text{ and } x \oplus y \in \{2^1, 2^2\}\},\end{aligned}$$

and

$$\begin{aligned}E_3 = \{&(x,y) \in V \times V \colon x_0 = y_0 = x_{t+1} = y_{t+1} = 0\\ &\text{and } x \oplus y = 2^k \text{ for } k \in [t+3, t+s]\} \cup\\ &\{(x,y) \in V \times V \colon x_0 = y_0 = 0, x_{t+1} = y_{t+1} = 1\\ &\text{and } x \oplus y = 2^k + 2^{k-1} \text{ for } k \in [t+3, t+s]\} \cup\\ &\{(x,y) \in V \times V \colon x_0 = y_0 = 0 \text{ and } x \oplus y \in \{2^{t+1}, 2^{t+2}\}\}.\end{aligned}$$

For convenience, we write $E_i(G)(= E_i)$ for $i \in \{1,2,3\}$ if we need to indicate the graph G. From the above definition, the binary string of a vertex in $LeTQ(s,t)$ is partitioned into three parts: s-part (i.e., the leftmost part with s bits), t-part (i.e., the middle part with t bits) and the rightmost bit. Accordingly, $LeTQ(s,t)$ contains 2^{s+t+1} vertices. Also, we have $|E_1| = 2^{s+t}$, $|E_2| = t2^{s+t-1}(= 2^s \times t2^{t-1})$ and $|E_3| = s2^{s+t-1}(= 2^t \times s2^{s-1})$. Thus, the total number of edges in $LeTQ(s,t)$ approaches one half of that in LTQ_{s+t+1} as s and t are larger. To be more precise, $|E(LeTQ(s,t))| = (s+t+2)2^{s+t-1} = (\frac{1}{2} + \frac{1}{2(s+t+1)})|E(LTQ_{s+t+1})|$. Fig. 1 depicts the locally exchanged twisted cube $LeTQ(1,3)$, where links with label "0" correspond to the edge set E_1, links