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Optimal online markdown and markup pricing policies with demand uncertainty

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ABSTRACT

In this paper, we study the dynamic pricing problem with demand uncertainty from the perspective of online algorithms and competitive analysis which eliminate the need for both the functional relationship between price and demand and the customer arrival rate. Assuming customer's reservation price falls in a closed interval, we prove the upper bound of any online policy for the HBL problem where the reservation price of a customer showing up later is always no more than the reservation price of a customer showing up earlier. And an optimal online markdown pricing policy whose competitive ratio matches the upper bound is proposed. For the symmetrical problem with LBH manner, we prove that there is no deterministic online policy whose competitive ratio is larger than the ratio of the lowest reservation price and the highest reservation price. For a special LBH problem with enough potential demand, we propose an optimal online markup policy dealing with not only how to price but also how much item to limit for each price.

1. Introduction

Consider a retailer holding a fixed inventory of summer apparel, new items are stocked in the spring and sold over several months. The retailer needs to decide when to adjust price based on the quantity sold out at prior price and how much to change the price dynamically. To optimize revenue, the retailer is better off (at least in the short run) discounting the stock than allowing it to perish. As a result, many sellers of perishable goods use a *markdown* pricing policy whereby an item is initially sold at full price and then subjected to deeper and deeper discounts until inventory is completely sold or the sale season ends, whichever comes first [1]. This dynamic pricing policy is commonly used by many vendors of seasonal, fashion, and perishable goods (see [2,3] and references therein).

Consider dynamic pricing strategies, there are substantial studies in revenue management (RM) literature (see [4,5] and references therein). A critical assumption made in most academic studies of RM problems is that the demand function (functional relationship between price and demand) or the customer arrival rate (e.g., Poisson process) is known to the decision maker. This assumption of "full information" endows the decision maker with knowledge that she/he does not typically possess in practice [6]. Without knowing the demand function perfectly, the pricing models in RM have largely employed the basic assumption that demand can be forecast with reasonable accuracy [5,7,8]. However, standard forecast methods rely heavily on demand history that is not necessarily available in the short life cycle environment, for example, retailers selling perishable fashion goods [9,10].

Although the consumers' reservation price on the item usually decreases over time in the field of fashion industry, evolving fashion trends generate great uncertainty in the number of customers who will consider purchasing these

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items [11]. This kind of phenomenon also appears in the theater field, apparel field and so on. Therefore, it is a very important task for the retailer to adjust price dynamically based on uncertain demand. We consider this dynamic price optimization problem from the perspective of *online strategy* (see [12] for an extensive discussion, [13] for a recent survey and [14] for an application in pricing problem). This strategy eliminates the need for both the functional relationship between price and demand and the customer arrival rate. The method is particularly suitable for decision makers who have to respond to events over time (customers visit the summer apparel one by one). To design a better or the best online strategy, *competitive analysis* is certainly useful in which an *imaginary adversary* is postulated that controls the input sequence. The goal of the adversary is to generate the *worst algorithmic performance* for the given online strategy. Specifically, after each step of the online strategy, the adversary selects the input so as to create the worst possible algorithmic performance. For a revenue-maximization problem P, we measure performance as the ratio of the objective revenue achieved by the online policy divided by the objective revenue that could be achieved by an optimal offline policy operating on the same input sequence. We now precisely define this performance measure, which is usually called the *competitive ratio*. Let Ω_A be the set of all possible input sequences to an online algorithm A. For $\forall I \in \Omega_A$, let $R_A(I)$ be the objective revenue achieved by A for input I and let $R_{OPT}(I)$ be the objective revenue achieved by an optimal offline algorithm. Then, we define $c_A = \inf_{I \in \Omega_A} \frac{R_A(I)}{R_{OPT}(I)}$ the competitive ratio of A. We say that a deterministic online algorithm A^* is the best one, if the competitive ratio of A^* is $c_{A^*} = \sup c_A$. Thus, the competitive ratio of an online algorithm is a guarantee on a certain level of performance.

The study most related with our work is the *single period* pricing models [15–18]. Studies of single period pricing models were motivated by research on production-pricing problems. In this field, most models assume that either the demand function or the customer's arriving rate is known. For example, Li [19] proposed a continuous time model where demand is a controlled Poisson process. Gallego and van Ryzin [20] developed a continuous time model for perishable products under the assumption of no inventory replenishment. Consider demand uncertainty, Ball and Queyranne [21] first analyzed the online booking policies within airline RM problem from the perspective of online strategy. In their paper, the classes of fares are fixed as exogenous variables and the fare class of request is also known to the airline. This assumption is adopted in most quantity-based RM models [5]. Moreover, in many price-based models, the possible price values are approximated to a finite set based on customer classes. For example, Feng and Gallego [22] investigated the optimal timing of a single price change from a given initial price to either a given lower or higher second price. They also discussed Markovian demand and fares [23]. Feng and Xiao [24,25] generalized the two-price model to consider risk preference and multiple prices. Chatwin [26] considered a continuous-time, dynamic pricing problem where prices have to be selected from a finite set of prices. The assumption on candidate price set is technical and in such a formulation the pricing policy would be handled [27]. While, in practice the decision maker cannot know the customer's reservation price whatever the price she/he pays for the product is, that is, the assumption about the price classes may not cover all possible kinds of customers. A nature way to introduce the price decision variable would be as a real number on some interval. Additionally, from a practical point of view, the times of price changing should be limited to no more than a fixed number, otherwise, there will be too much management fees and it is also difficult to deal with price adjusting frequently.

In practice, there are also many sellers of perishable goods using a *markup* pricing policy. For example, in the airline industry one airline usually raises the price of ticket based on the tickets sold out at prior price till the departure. In this field, most academic studies assume that the process of the passengers meets the low-before-high (LBH) manner, i.e., demand in lower reservation price class arrives earlier [28]. Thus, we study both markdown and markup price optimization problem from the perspective of online strategy in this paper. Our main results are summarized as follows. For the problem with HBL manner (customer with higher reservation price arrives earlier, HBL problem in short), we give an upper bound for any online markdown pricing policy and propose one optimal online policy whose competitive ratio matches the upper bound. For the general problem with LBH manner, LBH problem in short, we prove that there is no competitive online policy. For the special LBH problem where the potential demand is large enough, we design an optimal online markup pricing policy along with decisions on quantity limitation for each price.

The rest of this paper is organized as follows. In Section 2, we state the basic problem and describe some online information. The upper bound and an optimal online markdown policy are given for the HBL problem in Section 3. Section 4 deals with the LBH problem. Finally, conclusions are discussed in Section 5.

2. Problem definition and notation

The problem considered in this paper can be described as follows. One retailer of perishable goods has n unit item at the beginning and no inventory replenishment is permitted. The goods are perishable in the sense that they have to be sold before the end of sale season and have no value if they remain unsold. Therefore, the objective of the retailer is to maximize the revenue by adjusting the price dynamically. Before the potential customers arriving to shop, the retailer has to post a price f and each customer will buy one unit if her/his reservation price is at least f. We assume each customer buy one unit at most. And we assume that the reservation price of every customer falls in the closed interval $[\underline{f}, \overline{f}]$. From a practical point of view, like in the fields of theater, apparel, airline and so on, we assume the sequence of customers arriving is monotonous with their reservation prices. Under this monotonicity assumption, the reservation price of the earlier customer is no less than (or no higher than) that of the later one. Thus, we call this problem HBL problem (or