Information Processing Letters 115 (2015) 408–413

Contents lists available at ScienceDirect

Information Processing Letters

www.elsevier.com/locate/ipl

A greedy topology design to accelerate consensus in broadcast wireless sensor networks ☆

Massimo Vecchio [a,*], Roberto López-Valcarce [b]

[a] *CREATE-NET, Via alla Cascata 56/D, 38123 Trento, Italy*
[b] *Departamento de Teoría de la Señal y las Comunicaciones, Universidad de Vigo, 36310 Vigo, Spain*

ARTICLE INFO

Article history:
Received 19 February 2014
Received in revised form 18 November 2014
Accepted 18 November 2014
Available online 27 November 2014
Communicated by M. Chrobak

Keywords:
Average consensus
Range assignment
Topology control
Wireless multicast advantage
Graph algorithms

ABSTRACT

We present techniques to improve convergence speed of distributed average consensus algorithms in wireless sensor networks by means of topology design. A broadcast network is assumed, so that only the transmit power of each node can be independently controlled, rather than each individual link. Starting with a maximally connected configuration in which all nodes transmit at full power, the proposed methods successively reduce the transmit power of a chosen node in order to remove one and only one link; nodes are greedily selected either in order to yield fastest convergence at each step, or if they have the largest degree in the network. These greedy schemes provide a good complexity–performance tradeoff with respect to full-blown global search methods. As a side benefit, improving the convergence speed also results in savings in energy consumption with respect to the maximally connected setting.

1. Introduction

Average consensus, in the general framework of *networks of agents*, means reaching an agreement on the average state of all agents. Recently, much effort has been directed to the study of the average consensus problem in Wireless Sensor Networks (WSNs) (see [1] and the references therein), since distributed consensus algorithms only require iterative local information exchanges among neighboring nodes and the computation of weighted sums at each node. Potential applications include detection, estimation, reputation management, load balancing, control of autonomous agents, etc. [2].

One important issue regarding distributed average consensus algorithms in WSNs is convergence speed: reducing the convergence time results in fewer transmissions and therefore in energy savings. Approaches from the literature to speed up convergence can be classified into two groups. If the topology of the network is fixed, one can design the weights intervening in the consensus scheme in order to minimize convergence time [3–5]. On the other hand, if the network topology can somehow be altered, then additional flexibility is available, and the optimization can be performed over the topology as well as the weights [6–10]. Generally speaking, topology optimization is a very difficult combinatorial problem and different suboptimal approaches can be adopted. In [6] the convergence properties of different topology classes are theoretically analyzed on average, given the number of nodes of a general network. In [8] it is shown that, starting from a given topology, removing certain links can be beneficial in terms of convergence speed; this approach was later refined in [9]

☆ Research supported by the European Regional Development Fund (ERDF) and the Spanish Government (TEC2013-47020-C2-1-R COMPASS, CONSOLIDER-INGENIO 2010 CSD2008-00010 COMONSENS, TACTICA), and the Galician Regional Government (R2014/037, GRC2013/009 and AtlantTIC).

* Corresponding author. Tel.: +39 0461 408400; fax: +39 0461 421157.
E-mail addresses: massimo.vecchio@create-net.org (M. Vecchio), valcarce@gts.uvigo.es (R. López-Valcarce).

http://dx.doi.org/10.1016/j.ipl.2014.11.009
0020-0190/© 2014 Elsevier B.V. All rights reserved.

in order to judiciously remove and add links with the goal of speeding up the consensus algorithm while keeping energy consumption at bay. In WSNs with static nodes, topology control can be achieved by varying the transmit power, as considered in [7,10].

All of these approaches implicitly assume *unicast* pairwise communication[1]: each node can independently set the transmit power it allocates to communicate with each of its neighbors, *e.g.*, by using orthogonal signaling. However, in distributed consensus schemes the information that nodes need to send at a given iteration is the same for all of its neighbors, so that in WSNs it is possible to exploit the broadcast nature of the wireless channel, also known as Wireless Multicast Advantage (WMA) [11]: at each consensus iteration, each node may broadcast its state while its neighbors simultaneously listen, thus reducing the number of required transmissions. On the other hand, exploiting WMA while varying the transmit power of a given node affects the links to *all* of its neighbors, so that these cannot be independently controlled now.[2] This motivates specific topology optimization strategies that take this fact into account, since previous topology control schemes designed under the unicast assumption cannot be applied under these "broadcast communication" constraints.

This problem is related to the so-called *range assignment* (RA) problem in broadcast WSNs, usually oriented to other network-related goals (*e.g.* maintaining global connectivity [13]) and known to be difficult in general [14]. Our goal is to determine the transmit power for each node in a broadcast WSN in order to minimize the convergence time of a given distributed average consensus scheme. One issue featuring in such setting is the fact that, if the transmit powers of nodes i and j are different (non-homogeneous RA [13]), it may well happen that node i is out of the coverage range of node j whereas node j can listen to node i's transmissions; in other words, the underlying graph becomes *directed*. This has implications for consensus algorithms. Although reaching an agreement over a directed graph is easily achieved, the agreement value will be a *weighted* average of the agents' states, and the weights will depend on the topology. When the *unweighted* average is of interest, certain stringent requirements on the directed graph must be imposed (such as some sort of graph balancing [15]), which are generally difficult to enforce in practice. Hence, we focus on undirected graphs, for which reaching an agreement on the *unweighted* average by consensus algorithms is not a problem. To this end, we adopt a simple strategy by which nodes just ignore transmissions received from neighbors which are not within their own transmit range (in the previous example, node j would simply ignore packets received from node i), thus obtaining an undirected topology. With this framework, we start from a maximally connected setting (all nodes transmit at full power), and then proceed to iteratively reduce the power of one node at a time in a centralized greedy fashion in order to maximize the convergence rate.

As previous approaches to topology control [6–10], ours is a centralized scheme which can be run by a central entity after deployment and previously to the network becoming operative; after such step, network operation may become decentralized. Fully distributed topology control methods are desirable and should be the target of future research.

In Section 2 the network model and the basics of consensus schemes are presented. The proposed greedy algorithm for non-homogeneous RA is presented in Section 3. Simulation results and conclusions are provided in Sections 4 and 5.

2. Problem setting

2.1. Graph model

Consider a set $\mathcal{V}$ of randomly deployed nodes with indices $i \in \{1, \cdots, n\}$. Let d_{ij} be the distance between nodes i and j, and let $\mathcal{R} = \{r_i \in [0, r_{\max}],\ i = 1, \cdots, n\}$ be a set of connectivity radii (*i.e.* transmit ranges), with $r_{\max} > 0$ the maximum allowable range. We adopt a simple model by which a link between two nodes exists iff their distance does not exceed the transmit range of the transmitter, which can be controlled by setting the transmit power [10,13]. As discussed in Section 1, the edge set $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ is defined as

$$(i, j) \in \mathcal{E} \quad \Leftrightarrow \quad i \neq j, \quad d_{ij} \leq \min\{r_i, r_j\}. \tag{1}$$

In this way, the graph $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$ is undirected as desired. If $r_i = r$ for $i = 1, \cdots, n$ we recover the homogeneous RA over the standard Random Geometric Graph (RGG) model [16]. However, we allow for different transmit ranges at different nodes in order to add flexibility to the design.

The neighborhood of node i is defined as $\mathcal{N}_i = \{j : (i, j) \in \mathcal{E}\}$, and its degree (number of neighbors) is therefore $|\mathcal{N}_i|$. The graph Laplacian matrix $\boldsymbol{L}$ has elements

$$\boldsymbol{L}_{ij} = \begin{cases} |\mathcal{N}_i| & \text{if } i = j \\ -1 & \text{if } i \neq j \text{ and } j \in \mathcal{N}_i \\ 0 & \text{otherwise.} \end{cases} \tag{2}$$

By construction $\boldsymbol{L}$ is symmetric. Let $\lambda_1(\boldsymbol{L}) \leq \lambda_2(\boldsymbol{L}) \leq \cdots \leq \lambda_n(\boldsymbol{L})$ denote its ordered eigenvalues. Note that $\lambda_1(\boldsymbol{L}) = 0$ with corresponding eigenvector the $n \times 1$ all-ones vector $\mathbf{1}$. Moreover, $\lambda_2(\boldsymbol{L}) > 0$ iff $\mathcal{G}$ is connected [16]. $\lambda_2(\boldsymbol{L})$ is known as the *algebraic connectivity* of the graph.

2.2. Average consensus algorithms

Let $\boldsymbol{x}(0) = [x_1(0) \cdots x_n(0)]^T \in \mathbb{R}^n$ denote the vector of initial node measurements. The goal of the average consensus algorithm is to have all nodes compute the average $\bar{x} = \frac{1}{n}\mathbf{1}^T\boldsymbol{x}(0)$, iteratively and in a distributed fashion (thus node i can only communicate with nodes in $\mathcal{N}_i$). Distributed linear iterations [3] take the form $\boldsymbol{x}(k) = \boldsymbol{W}\boldsymbol{x}(k-1)$, where

---

[1] As an exception, in [10, Sec. V] a broadcast scheme is considered, but the transmit power is constrained to be equal for all nodes in the network, in contrast with the approach proposed in the present work.

[2] Transmissions in a broadcast WSN should be coordinated at the MAC layer in order to avoid collisions and align the listening and transmitting nodes, for instance by implementing some suitable time-synchronization protocol [12].