Available online at www.sciencedirect.com
ScienceDirect
Journal of Algorithms 62 (2007) 19–46
Journal of Algorithms
www.elsevier.com/locate/jalgor

Solving NP-hard semirandom graph problems in polynomial expected time ☆

Amin Coja-Oghlan

Humboldt-Universität zu Berlin, Institut für Informatik, Unter den Linden 6, 10099 Berlin, Germany

Received 20 February 2003

Available online 25 September 2004

Abstract

The aim of this paper is to present an SDP-based algorithm for finding a sparse induced subgraph of order $\Theta(n)$ hidden in a semirandom graph of order n. As an application we obtain an algorithm that requires not more than $O(n)$ random edges in order to k-color a semirandom k-colorable graph within polynomial expected time, thereby extending results of Feige and Kilian [J. Comput. System Sci. 63 (2001) 639–671] and of Subramanian [J. Algorithms 33 (1999) 112–123].
© 2004 Elsevier Inc. All rights reserved.

1. Introduction and results

1.1. Semirandom graph problems

NP-hardness of approximation results are known for many fundamental optimization problems. For instance, approximating the size of a maximum independent set or the chromatic number of a graph within $n^{1-\varepsilon}$, $\varepsilon > 0$ constant, is NP-hard [5,12]. Also, it is NP-hard to 4-color 3-colorable graphs [15]. Thus, since there is not much hope for efficient algo-

☆ Research supported by the Deutsche Forschungsgemeinschaft (grant DFG FOR 413/1-1). An extended abstract version of this paper has appeared in the Proc. of RANDOM 2002, Lecture Notes in Comput. Sci., vol. 2483, Springer, pp. 139–148.

E-mail address: coja@informatik.hu-berlin.de.

0196-6774/$ – see front matter © 2004 Elsevier Inc. All rights reserved.
doi:10.1016/j.jalgor.2004.07.003

rithms that perform well in the worst case, it is natural to ask for efficient algorithms that can handle "average" instances.

But how can we model "average" instances, e.g., of the graph coloring problem or the maximum independent set problem? One could take the binomial random graph $\mathcal{G}(n, p)$ for a first answer. Needless to say, the algorithmic theory of random graphs has become an important field of investigation [9]. However, algorithms for random graph problems tend to make extensive use of special properties of random graphs, such as the distribution of the vertex degrees, spectral properties etc. But what if the "average" instances we have in mind do not satisfy such "typical" properties?

Semirandom models may provide an answer. Instances of semirandom graph problems are made up of a random share and a worst case part added by an adversary. The smaller the random share is, the more the semirandom model resembles the worst case, and the harder the according algorithmic problem becomes. Moreover, algorithms that rely on typical properties of random graphs will in general be useless in the semirandom case, because of the adversary. Conversely, one may conclude that algorithms that perform well on semirandom instances are comparatively robust. Thus, the aim of studying semirandom graph problems is to achieve algorithms that apply to a wider class of input distributions.

The first semirandom models have been suggested by Blum and Spencer [2], who have studied the problem of coloring a semirandom k-colorable graph. For instance, Blum and Spencer study the following model: the vertex set $V = \{1, \ldots, n\}$ is partitioned randomly into k disjoint classes $V_1, \ldots, V_k$ of cardinality n/k each, where k is a fixed integer. Then, each edge $\{v, w\}$ joining vertices in different classes is included into G_0 with probability p, independently of all others. Hence, the random share G_0 of the instance is a random k-colorable graph. Finally, an adversary may add further edges $\{v, w\}$ as above, thereby completing the instance G. By construction, $V_1, \ldots, V_k$ is a k-coloring "planted" in G. We say that G_0 has some property $\mathcal{P}$ *with high probability* if the probability that G_0 has property $\mathcal{P}$ tends to 1 as $n \to \infty$. Moreover, we say that G has a certain property $\mathcal{P}$ with high probability, if with high probability the random share G_0 of G is such that regardless of the behavior of the adversary the instance G has property $\mathcal{P}$. Blum and Spencer propose a combinatorial algorithm that k-colors the semirandom k-colorable graph with high probability provided that $np \geqslant n^{\alpha_k}$ for a certain constant $\alpha_k > 0$.

Using semidefinite programming techniques, Feige and Kilian [6] have improved on the results of Blum and Spencer considerably. The algorithm of Feige and Kilian k-colors a semirandom k-colorable graph G with high probability, provided that $p \geqslant (1+\varepsilon)k(\ln n)/n$. On the other hand, they prove that if $p \leqslant (1-\varepsilon)(\ln n)/n$, then there is no polynomial time algorithm that k-colors G with high probability, unless NP $\subset$ BPP.

Moreover, Feige and Kilian have studied the following semirandom model for the maximum independent set problem. Let $0 < \alpha < 1$ be a fixed number, and let $V = \{1, \ldots, n\}$ be a set of n vertices. First, a set $S \subset V$ of $\lceil \alpha n \rceil$ vertices is chosen uniformly at random. Then, each S-$V \setminus S$-edge is included into the random share G_0 of the instance with probability p independently of all other edges. Note that G_0 is just a random bipartite graph. Finally, the adversary may add further $(V \setminus S)$-V-edges (i.e., edges not joining two vertices in S) to complete the instance G. Clearly, the set S remains an independent set "planted" in G. Feige and Kilian show that if $p \geqslant (1+\varepsilon)\ln(n)/(\alpha n)$ for an arbitrarily small constant $\varepsilon > 0$, then it is possible to recover the hidden set S in polynomial time with high prob-

Download English Version:

https://daneshyari.com/en/article/429336

Download Persian Version:

https://daneshyari.com/article/429336

Daneshyari.com