Journal of Computational Science 5 (2014) 119–125

Contents lists available at ScienceDirect

Journal of Computational Science

journal homepage: www.elsevier.com/locate/jocs

Mathematical modelling and parameter optimization of pulsating heat pipes

Xin-She Yang[a,*], Mehmet Karamanoglu[a], Tao Luan[b], Slawomir Koziel[c]

[a] School of Science and Technology, Middlesex University, London NW4 4BT, UK
[b] School of Energy and Power Engineering, Shandong University, Jinan, China
[c] School of Science and Engineering, Reykjavik University, IS-103 Reykjavik, Iceland

ARTICLE INFO

Article history:
Received 4 October 2012
Received in revised form
25 November 2013
Accepted 13 December 2013
Available online 6 January 2014

Keywords:
Asymptotic
Firefly algorithm
Heat transfer
Mathematical modelling
Optimization
Oscillations
Pulsating heat pipe
Two-phase flow

ABSTRACT

Proper heat transfer management is important to key electronic components in microelectronic applications. Pulsating heat pipes (PHP) can be an efficient solution to such heat transfer problems. However, mathematical modelling of a PHP system is still very challenging, due to the complexity and multiphysics nature of the system. In this work, we present a simplified, two-phase heat transfer model, and our analysis shows that it can make good predictions about startup characteristics. Furthermore, by considering parameter estimation as a nonlinear constrained optimization problem, we have used the firefly algorithm to find parameter estimates efficiently. We have also demonstrated that it is possible to obtain good estimates of key parameters using very limited experimental data.

1. Introduction

A pulsating heat pipe is essentially a small pipe filled with both liquid and vapour and the internal diameter of the heat pipe is at the capillary scale [1,2]. The liquids in the pipe can form segments or plugs, between vapour segments. When encountering heat, part of the liquid many evaporate and absorb some heat, thus causing a differential pressure and driving the movement of the plugs. When vapour bubbles meet a cold region, some of the vapour may condensate, and thus releasing some heat. The loop can be open or closed, depending on the type of applications and design. This continuous loop and process will form an efficient cooling system if designed and managed properly for a given task. Therefore, such systems have been applied to many applications in heat exchanger, space applications and electronics, and they can potentially have even wider applications [43]. On the other hand, the emergence of nanotechnology and the steady increase of the density of the large-scale integrated circuits have attracted strong interests in modelling heat transfer at very small scales, and the heat management of microdevices has become increasingly important for next generation electronics and miniaturization.

Both loop heat pipes (LHP) and pulsating heat pipes (PHP) may provide a promising solution to such challenging problems, and thus have attracted renewed attention in recent years [1,4,7,12,13,24,25,42]. In many microelectronic applications, conventional solutions to heat management problems often use fans, heat exchangers and even water cooling. For examples, the fan-driven air circulation system in desktop computers and many laptops have many drawbacks such as bulky sizes and potential failure of mechanical, moving parts. In contrast, loop heat pipes have no mechanical driving system, and heat circulation is carried out through the pipes, and thus such LHP systems can be very robust and long-lasting. In addition, miniaturization and high-performance heat pipes systems are being developed [11,15,18,26,39,43]. Simulation tools and multiphase models have been investigated [16,17]. All these studies suggested that LHP systems can have some advantages over traditional cooling systems.

A PHP system may often look seemingly simple; however, its working mechanisms are relatively complex, as such systems involve multiphysics processes such as thermo-hydrodynamics, two-phase flow, capillary actions, phase changes and others. Therefore, many challenging issues still remain unsatisfactorily modelled. There are quite a few attempts in the literature to model a PHP system with various degrees of approximation and success.

In this paper, we will use a mathematical model based on one of the best models [24,25], and will carry out some mathematical

* Corresponding author. Tel.: +44 2084112351.
E-mail address: x.yang@mdx.ac.uk (X.-S. Yang).

1877-7503/$ – see front matter © 2013 Elsevier B.V. All rights reserved.
http://dx.doi.org/10.1016/j.jocs.2013.12.003

analysis and highlight the key issues in the state of the art models. In this paper, we intend to achieve two goals: to present a simplified mathematical model which can reproduce most characteristics of known physics, and to provide a framework for estimating key parameters from a limited number of measurements. The rest of the paper is organized as follows: we first discuss briefly design optimization and metaheuristic algorithms such as firefly algorithm, we then outline the main multiphysics processes in the mathematical models. We then solve the simplified model numerically and compared with experimental data drawn from the literature. After such theoretical analysis, we then form the parameter estimation as an optimization problem and solve it using the efficient firefly algorithm for inversely estimating key parameters in a PHP system. Finally, we highlight the key issues and discuss possible directions for further research.

2. Design optimization of heat pipes

The proper design of pulsating heat pipes is important, so that the heat in the system of concern can be transferred most efficiently. This also helps to produce designs that use the least amount of materials and thus cost much less, while lasting longer without the deterioration in performance. Such design tasks are very challenging, practical designs tend to be empirical and improvements tend to be incremental. In order to produce better design options, we have to use efficient design tools for solving such complex design optimization problems. Therefore, metaheuristic algorithms are often needed to deal with such problems.

2.1. Metaheuristics

Metaheuristic algorithms such as the firefly algorithm and bat algorithm are often nature-inspired [28,31], and they are now among the most widely used algorithms for optimization. They have many advantages over conventional algorithms, such as simplicity, flexibility, quick convergence and capability of dealing with a diverse range of optimization problems. There are a few recent reviews which are solely dedicated to metaheuristic algorithms [28,29,34]. Metaheuristic algorithms are very diverse, including genetic algorithms, simulated annealing, differential evolution, ant and bee algorithms, particle swarm optimization, harmony search, firefly algorithm, cuckoo search, flower algorithm and others [29,30,33,36].

In the context of heat pipe designs, we have seen a lot of interests in the literature [5,7]. However, for a given response, to identify the right parameters can be considered as an inverse problem as well as an optimization problem. Only when we understand the right working ranges of key parameters, we can start to design better heat-transfer systems. To our knowledge, this is the first attempt to use metaheuristic algorithms to identify key parameters for given responses. We will use the firefly algorithm to achieve this goal.

2.2. Firefly algorithm

Firefly algorithm (FA) was first developed by Xin-She Yang in 2008 [28,30] and is based on the flashing patterns and behavior of fireflies. In essence, FA uses the following three idealized rules:

1. Fireflies are unisex so that one firefly will be attracted to other fireflies regardless of their sex.
2. The attractiveness is proportional to the brightness and they both decrease as their distance increases. Thus for any two flashing fireflies, the less brighter one will move towards the brighter one. If there is no brighter one than a particular firefly, it will move randomly in the form of a random walk.
3. The brightness of a firefly is determined by the landscape of the objective function.

As a firefly's attractiveness is proportional to the light intensity seen by adjacent fireflies, we can now define the variation of attractiveness β with the distance r by

$$\beta = \beta_0 e^{-\gamma r^2}, \tag{1}$$

where β_0 is the attractiveness at $r = 0$.

The movement of a firefly i attracted to another more attractive (brighter) firefly j is determined by

$$\boldsymbol{x}_i^{t+1} = \boldsymbol{x}_i^t + \beta_0 e^{-\gamma r_{ij}^2}(\boldsymbol{x}_j^t - \boldsymbol{x}_i^t) + \alpha\, \boldsymbol{\epsilon}_i^t, \tag{2}$$

where the second term is due to the attraction. The third term is randomization with α being the randomization parameter, and $\boldsymbol{\epsilon}_i^t$ is a vector of random numbers drawn from a Gaussian distribution or uniform distribution at time t. If $\beta_0 = 0$, it becomes a simple random walk. Furthermore, the randomization $\boldsymbol{\epsilon}_i^t$ can easily be extended to other distributions such as Lévy flights.

The Lévy flight essentially provides a random walk whose random step size s is drawn from a Lévy distribution

$$Lévy \sim s^{-\lambda} \quad (1 < \lambda \le 3), \tag{3}$$

which has an infinite variance with an infinite mean. Here the steps essentially form a random walk process with a power-law step-length distribution with a heavy tail. Some of the new solutions should be generated by Lévy walk around the best solution obtained so far; this will speed up the local search. Lévy flights are more efficient than standard random walks [29].

Firefly algorithm has attracted much attention [3,10,23,35]. A discrete version of FA can efficiently solve non-deterministic polynomial-time hard, or NP-hard, scheduling problems [23], while a detailed analysis has demonstrated the efficiency of FA for a wide range of test problems, including multiobjective load dispatch problems [3,32]. Highly nonlinear and non-convex global optimization problems can be solved using firefly algorithm efficiently [9,35]. The literature of firefly algorithms have expanded significantly, and Fister et al. provided a comprehensive literature review [8].

3. Mathematical model for a PHP system

3.1. Governing equations

Mathematical modelling of two-phase pulsating flow inside a pulsating heat pipe involves many processes, including interfacial mass transfer, capillary force, wall shear stress due to viscous action, contact angles, phase changes such as evaporation and condensation, surface tension, gravity and adiabatic process. All these will involve some constitutive laws and they will be coupled with fundamental laws of the conservation of mass, momentum and energy, and thus resulting in a nonlinear system of highly coupled partial differential equations. Consequently, such a complex model can lead to complex behaviour, including nonlinear oscillations and even chaotic characteristics [14,19,20,24,27,41].

A mathematical model can have different levels of complexity, and often a simple model can provide significant insight into the working mechanism of the system and its behaviour if the model is constructed correctly with realistic conditions. In most cases, full mathematical analysis is not possible, we can only focus on some aspects of the model and gain some insight into the system.