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Probing and shaping the information transfer of noise-perturbed complex networks via Markov Chain analysis

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A B S T R A C T

We demonstrate a recursive computational procedure based on the distributions of first passage time on Markov Chains that can mathematically characterize noise-driven processes in complex networks. Considering examples of both real (Enron email) and artificial (Ravasz-Barabasi) networks perturbed by noise using Monte Carlo simulations, our method accurately recovers the percentages that information will be transferred to the intended receivers. The paradigm reported here captures and provides explanation to the recent results of Czaplicka et al. (Nature Sci. Rep. 2013) showing that the presence of noise can actually enhance the transfer of information in a hierarchical complex network. Finally, we illustrate how adaptive thresholding guided by our developed procedure can be used to engineer or shape the dynamic range of networks operating in a noisy environment.

1. Introduction

There has been a surge in interest in the field of complex networks in the recent years particularly after the discoveries made by Watts and Strogatz on the nature of small world networks [1] as well as Albert and Barabasi [2] in the characterization of how the topology of scale-free networks evolves. In particular, the role of complex networks as systems for information processing and transfer has also been investigated and applied to both artificial [3–5] and real networks [6,4]. The field of random walks on networks, specifically on how passage times are affected by different topologies, have also gained much interest in the recent years, even though the study of random walks and Markov Chains on regular networks have been studied for decades [7].

The applications of passage times are varied, ranging from search applications in the web [8], to diffusion reactions [9] as well as models of biological pathways for protein folding [10]. Previously, Condamin et al. [11] found that random walk passage times on scale-free networks can be compared to that of regular networks by using an alternative fractal dimension of the network. Lau et al. [12] demonstrated that the distribution of first passage time can be obtained using asymptotic analysis and Hwang et al. [13] also showed that in scale-free networks, the global mean passage time showed different scaling behaviors depending on both their spectral dimension and their scale-free degree exponent. However, these analyses are only applicable for the specific case of unbiased random walk with uniform probability of transition. In the case of the biased random walk, Fronczak et al. [14] analyzed the performance of mean passage time of a type of biased random walk which favored high degree nodes and concluded that the passage time for specific sender-receiver node pairs depended highly on the degree of the final target node. A biased random walk was also shown by Zlatic et al. [15] to be capable of detecting various communities within the network.

In a recent paper, Czaplicka et al. showed [16] that the addition of noise into an information transmission system based around the topology of a hierarchical network could enhance the efficiency of the network. This differs from the analysis previously done by Fronczak et al. since it incorporates topological noise that randomly causes edges to be rewired to different nodes. The combination of topological noise and random packet movement was shown to give rise to a resonance like effect when their parameters are being manipulated. This stochastic resonance-like effect [17,18]

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is expected to generate profound interest as the applications are wide-ranging. For example, it could be shown the addition of random noise could in fact improve the efficiency of search algorithms in the dynamically changing World Wide Web [19], enhance chemical reactions in metabolic networks [20] or enhance the rate of information spread on fast growing social networks. [21]

In this paper, we demonstrate that the system described by Czaplicka et al. [16] using Monte Carlo simulation can in fact be treated as a biased random walk on a lattice which can be mathematically handled using Markov Chains. The advantage of using Markov Chain based analysis is that it yields an exact result when computed as compared to Monte Carlo which translates to:

(1) Removal of the need to randomise the packet movement thus allowing incorporation of the packet noise into the numerical calculations;
(2) Possible removal of the need to randomise the edge rewiring which scales with the number of edges in the network; and
(3) Straightforward discernment of the behaviour of packets moving from selected sender receiver pairs.

In contrast with the methods demonstrated by Condamin et al. [11] (using pseudo Green's function) as well as Lau et al. [12] (considering the degree of the destination node and using asymptotic analysis), the method we formulated here applies to any kind of biased random walk which have unequal transition probabilities and any kind of network with known adjacency matrix. Furthermore, Markov Chain analysis allows us to clearly discern the mechanism through which the stochastic resonance-effect arises and conduct analysis of the distinct sender-receiver pairs generated in the network, a procedure that is computationally expensive and less accurate if attempted through Monte Carlo simulation. We further extend the discussion to describe how the procedure that we developed can be used to engineer the dynamic range of networks in a noisy environment via adaptive thresholding.

The paper is divided as follows: Section 2 provides a detailed description of the packet navigation algorithm being analysed. Section 3 describes the recursive computational procedure based on Markov Chains that we developed and briefly compares the computational complexity between the procedure with that of a similar procedure using Monte Carlo simulations. Section 4 discusses the results obtained from this Markov Chain analysis in networks exposed to varying noise. Section 5 illustrates an application of adaptive thresholding that is able to engineer the required performance for information transfer for different noise levels. Finally, Section 6 provides a summary of the findings and concludes the paper.

2. Packet navigation algorithm

In this section, we describe the packet navigation algorithm introduced by Czaplicka et al. [16] that we will be analysing throughout this paper. At the start of packet transmission, the set of nodes (V) in the network $G(V, E)$ are divided into various communities. Nodes within the community are understood to have a higher probability of having closer proximity to one another and we denote the set of nodes that are in the same community as the sender node r as $\mathbf{c}_r$. N_p packets containing information is subsequently transmitted through a network $G(V, E)$. Each packet is randomly assigned a sender s and receiver r node within the network and move across the network in the following manner:

```
1   Definition of terms;
2   Given a packet is currently at node i at time t;
3   q ∈ [0, 1] is a random probability value;
4   N(i) is the set of nodes directly connected to i;
5   c_r is the set of nodes in the community of receiver r and;
6   T_max is the threshold time for each packet to complete the delivery.;
7   Packet navigation algorithm;
8   While t ≤ T_max do
9     if i = r then
10      goto 22;
11    if r ∈ N(i) then
12      packet has probability (1 − q) to move to r and goto 22;
13      otherwise goto 14;
14    if c_r ∩ N(i) ≠ ∅ then
15      packet has probability (1 − q) to move randomly to any node in c_r,
    increment t and and goto 8;
16      otherwise goto 17;
17    else
18      packet moves randomly to a node in N(i);
19      increment t and goto 8;
20    end
21  end
22  if i = r then
23    record delivery time T = t
24  else
25    packet delivery failed
26  end
```

The probability q illustrated here acts as packet noise, a larger value of q essentially increases the randomness of the packet movement and decreases the likelihood of packets moving directly to the receiver node or to nearby community nodes.

The system is now subjected to a second type of noise, topological noise denoted as p. Topological noise is added in this manner; each edge in E is given a probability $p \in [0, 1]$ to be rewired to a different target node. The resulting modified graph $G' = (V, E')$ would therefore have a different structure from that of G. Importantly, the community information $\mathbf{c}_r$ is not updated to account for the new community structure. This results in a mismatch between the information possessed by the packets and the actual structure of G', and consequently, packets could preferably move to dummy community nodes which may not lead them closer to r.

The transmission efficiency of packet transmission system could subsequently measured using the following 2 metrics:

(a) $P\% - N_{success}/N_p$ - the percentage of packets transmitted that successfully arrive at their respective receiver nodes within the threshold time, T_{max}.
(b) $T_{inv} = 1/N_p \sum_{N_p} 1/T$ - the mean of the inverse time taken for each packet to arrive at its receiver node. Note that if the packet delivery failed, we set $T = \infty$, that is $1/T = 0$.

3. Recursive computation of packet navigation algorithm using Markov Chain analysis

For the case $q = 1$, i.e. maximum packet noise, we noted that the described procedure is a simple random walk on a network with equal transition probabilities of moving to neighboring nodes. For all other cases where $0 \leq q < 1$, the packet movement is a biased random walk with higher probabilities of moving towards r or any of its community nodes.

The location of each packet at time t can therefore be represented by the states of a Markov Chain. The measured packet delivery time T is therefore a random variable known in Markov Chain literature as the first passage time (FPT) between the sender and receiver nodes and the mean first passage time (MFPT) is the FPT averaged over a large number of packets. The averaged value of MFPT for all the possible combinations of s and r is also known as the global mean first passage time (GMFPT)[22] of the network and indeed the performance of these parameters have been