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# Near-pole polar diagram of objects and duality

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ABSTRACT

The polar diagram of a set of points in a plane and its extracted dual *EDPD* were recently introduced for static and dynamic cases. In this paper, the near-pole polar diagram *NPPD* for a set of points is presented. This new diagram can be considered as a generalization of the polar diagram and has applications in several communication systems and robotics problems. After reviewing the *NPPD* of points, we solve the problem for a set of line segments and simple polygons in optimal time $\Theta(n \log n)$, where $n$ is the number of line segments or polygon vertices. We introduce duality for the *NPPD* of points and identify some applications.



## 1. Introduction

Solutions to visibility and many other important problems in computational geometry require some type of angle processing of the data input. In this paper we introduce a relation between some visibility problems and a new plane partitioning process we call a near-pole polar diagram (*NPPD*). The problems in question occur in communication and radio systems, antenna arrangement and target detection. The approach proposed can also be applied to illumination problems, robot vision, hidden surface removal, collision detection and decorative patterning. *NPPD* is a generalization of the Voronoi diagram and is described in Section 3.

One of the most fundamental concepts in computational geometry is the Voronoi diagram, and its algorithms and applications have been studied extensively [6,8]. This concept has also been generalized in a variety of fields by replacing the standard Euclidean distance with other metrics such as the $L_p$ distance [11], weighted distances [4,7], the power distance [5], and a skew distance [2]. Using the Euclidean distance, the $\beta$-skeleton and proximity graphs have been defined and are used to solve other problems [1,3]. As stated later, these concepts can be redefined using a polar diagram for application in angle-related problems. As the solution to many problems in computational geometry requires some type of angle processing of the input, some other generalizations of the Voronoi diagram based on angles have been studied [9,10,12,13]. Grima et al. proposed a new locus approach for problems with angle processing, called the polar diagram [9]. For any position $q$ in a plane (represented by a point), the region in which $q$ lies belongs to the site with the smallest polar angle. Use of the polar diagram principle can help in solving some important problems that require angle processing in computational geometry. Grima et al. proved a preprocessing step using a polar diagram can speed up the process for many problems in computational geometry [9]. Applications include the convex hull, visibility problems and path-planning problems.

Sadeghi et al. introduced a dual of the polar diagram, the near-pole polar diagram for a set of points, and described some properties and applications [12,13]. Sadeghi et al. also solved a dynamic polar diagram in which new sites can be added to or removed from the point set[14,15]. In the following section we review some concepts and properties needed in subsequent sections. In Section 3, the *NPPD* of points is briefly introduced [13]. In Section 4, the *NPPD* of line segments and simple polygons is obtained. We define the duality of the *NPPD* for points in Section 5 and present an algorithm to find it. Section 6 concludes and identifies some open problems.

## 2. Polar diagram and its extracted dual (*EDPD*)

The polar angle of point $p$ with respect to $s_i$, denoted by $ang_{s_i}(p)$, is the angle formed by the positive horizontal line from $p$ and the straight line linking $p$ and $s_i$ (Fig. 1a).

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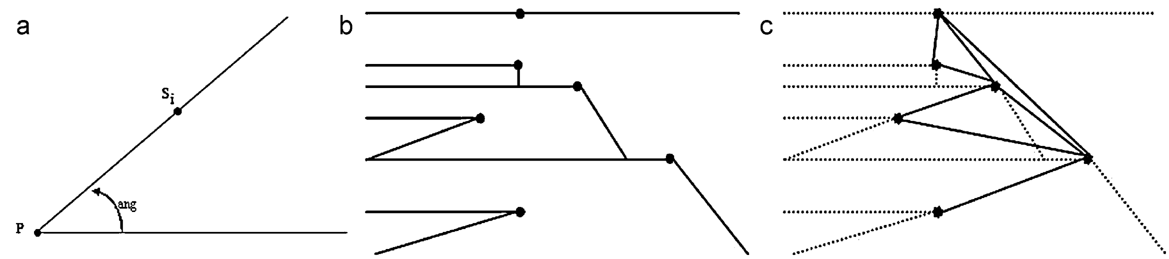


**Fig. 1.** (a) Polar angle, (b) polar diagram of six points and (c) corresponding extracted dual of the polar diagram (*EDPD*).

Given a set $S$ of $n$ sites (fixed points) in the plane, the locus of points with a smaller positive polar angle with respect to $s_i \in S$ is called the polar region of $s_i$. Thus,

$$\mathcal{P}_S(s_i) = \{(x, y) \in E^2 | ang_{S_i}(x, y) < ang_{S_j}(x, y); \forall j \neq i\}.$$

The plane is divided into different regions in such a way that if the point $(x, y) \in E^2$ lies in $\mathcal{P}_S(s_i)$, $s_i$ is the first site found when performing an angular scan starting from $(x, y)$. The resulting diagram is called a polar diagram. It is possible to draw an analogy between this angular sweep and the behavior of a radar [10]. Fig. 1b depicts the polar diagram of a set of points in a plane. Grima et al. proved that for a given set $S$ of $n$ points in a plane, the polar diagram of $S$ can be computed in time $\Theta(n\log n)$ using an incremental method [9].

Sadeghi et al. defined a dual of the polar diagram for a set of points [12]. They also defined another graph named the extracted dual of the polar diagram (*EDPD*) (Fig. 1c) and presented an optimal algorithm for finding *EDPD* in time $\Theta(n)$ for sorted sites and $\Theta(n \log n)$ for unsorted sites.

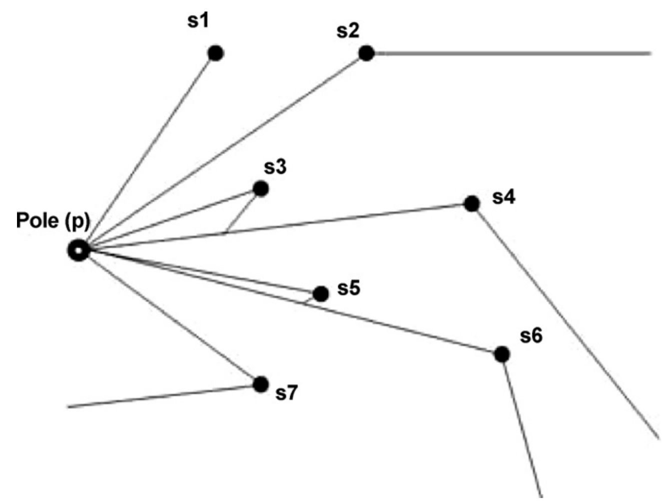


**Fig. 2.** Near-pole polar diagram of points.

## 3. Near-pole polar diagram (*NPPD*) for a set of points

In the classic definition of the polar diagram, the pole lies on the left-hand side of the plane at $-\infty$ [12,9]. In *NPPD*, it is assumed that the pole is located on the left-hand side of the sites and close to them. Thus, it can be concluded that this problem is a generalization of the polar diagram. This means that the problem is applicable to more situations; for example, the pole can be considered as the center of vision (eye) of a robot.

In addition to the given point sites, a point $p$ in the plane is also given as a pole and partitioning of the plane will depend on the position of $p$. Site $s_i$ and the arbitrary point $x$ are also given in the plane. Throughout this paper, we consider the angle formed by the lines $ps_i$ and $xs_i$ as the polar angle. Without loss of generality, assume that pole $p$ is located on the left-hand side of the sites. Fig. 2 shows an example of *NPPD* for seven points with respect to pole $p$.

In short, a near-pole polar diagram can be described as follows. Initially there is a radar at each of the point sites looking at the pole. These simultaneously start rotating counterclockwise and scan the periphery. The region in the plane observed by radar $p_i$ before other radars is called the region of $p_i$.

To draw *NPPD* for a set of points, an assumption is made to simplify the problem. It is assumed that the line of view for each site blocks that for any other site that intercepts it later. Fig. 3 depicts two sites $s_1$ and $s_2$, the pole $p$ and a point $x$ in the plane as the input. In partitioning of the plane, since $p\hat{s_1}x < p\hat{s_2}x$, $x$ belongs to the region of $s_1$. In this figure, the line segment $ps_2$ blocks the line of vision for $s_1$.

**Assumption 1.** The line of vision for each site blocks that for any other site that intercepts it later.

Sadeghi et al. presented is an algorithm for drawing the *NPPD* of points in optimal $\Theta(n \log n)$ time [13]. The slope of the line segments $s_ip : i = 1, \ldots, n$ is first computed and the values are sorted. Then a ray starting at pole $p$ rotates clockwise around pole $p$ and sweeps the plane. This algorithm runs in $\Theta(n \log n)$ time.

## 4. *NPPD* of geometric objects

*NPPD* of a set of points is reviewed in the previous section. In this section, two algorithms to find *NPPD* of a set of line segments and simple polygons in a plane are presented.

Given a set of line segments or a set of polygons, first the *NPPD* of endpoints and vertices is found using Algorithm 1 from [13], then some pieces of the edges in the diagram are removed. As for

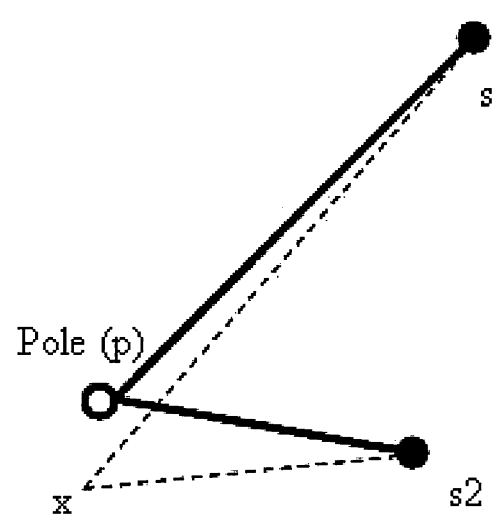


**Fig. 3.** Assumption: the line segment $ps_2$ blocks the line of vision for $s_1$.