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Belief revision within fragments of propositional logic ☆,☆☆

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ABSTRACT

Belief revision has been extensively studied in the framework of propositional logic, but just recently revision within fragments of propositional logic has gained attention. Hereby it is not only the belief set and the revision formula which are given within a certain language fragment, but also the result of the revision has to be located in the same fragment. So far, research in this direction has been mainly devoted to the Horn fragment of classical logic. Here we present a general approach to define new revision operators derived from known operators, such that the result of the revision remains in the fragment under consideration. Our approach is not limited to the Horn case but applicable to any fragment of propositional logic where the models of the formulas are closed under a Boolean function. Thus we are able to uniformly treat cases as dual Horn, Krom and affine formulas, as well.

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1. Introduction

Belief revision is a central topic in knowledge representation and reasoning. Belief revision consists in incorporating a new belief, changing as few as possible of the original beliefs while preserving consistency. Within the symbolic frameworks, where the beliefs are represented by logical formulas, the AGM paradigm [2] dedicated to the revision of theories, became a standard which provides rational postulates any reasonable revision operator should satisfy. Katsuno and Mendelzon [3], when unifying semantic revision approaches, reformulated these postulates where a theory is represented by a propositional formula. Moreover they proposed a representation theorem that characterizes revision operations in terms of total pre-orders over interpretations.

Belief revision has been extensively studied within the framework of propositional logic and numerous concrete belief revision operators have been proposed according to either semantic or syntactic points of view, for example [4–6]. Moreover, complexity results have been obtained [6–8]. However, as far as we know, only few works have focused on belief revision within the framework of *fragments of propositional logic*, except for the Horn case [9–11].

The study of belief change within language fragments is motivated by two central observations:

- In many applications, the language is restricted a priori. For instance, a rule-based formalization of expert knowledge is much easier to handle for standard users. In case users want to revise some rules, they indeed expect that the outcome is still in the easy-to-read format they are used to.

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☆☆ This is an extended and enhanced version of [1].

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- Many fragments of propositional logic allow for efficient reasoning methods. Suppose an agent who frequently has to answer queries about his beliefs. This should be done efficiently thus the beliefs are stored as a formula known to be in a tractable class. In case the beliefs of the agent are undergoing a revision, it is desired that the result of such an operation yields a formula in the same fragment. Hence, the agent still can use the dedicated solving method he is equipped with for this fragment. In case such changes are performed rarely, we do not bother whether the revision itself can be performed efficiently, but it is more important that the outcome can still be evaluated efficiently.

Katsuno and Mendelzon [3] provided an elegant framework for belief revision where a theory is finitely represented by a formula. Throughout the paper we will follow Katsuno and Mendelzon's view of revision, since fragments of logic are more directly expressed in this approach, whereas the AGM approach would require fragment-tailored definitions for belief sets. In order to investigate how known operators can be refined such that they work properly within a language fragment $\mathcal{L}'$ we have to deal with the following obstacle: given formulas $\psi, \mu \in \mathcal{L}'$, there is no guarantee that the outcome of a revision, $\psi \circ \mu$, remains in $\mathcal{L}'$ as well. Let, for example, $\psi = a \wedge b$ and $\mu = \neg a \vee \neg b$, be formulas expressed in conjunctive normal form (CNF) with Horn clauses (at most one positive literal), revising ψ by μ using Dalal's revision operator [4] does not remain in the Horn language fragment since $(a \vee b) \wedge (\neg a \vee \neg b)$ belongs to the result of the revision. The natural question arises whether there exist refinements $\star$ of $\circ$ such that $\psi \star \mu \in \mathcal{L}'$ always holds, but properties of $\circ$ are retained whenever possible. For instance, for such a refined operator it seems reasonable that $\psi \star \mu$ is equivalent to $\psi \circ \mu$ whenever $\psi \circ \mu$ already yields a result on the desired fragment $\mathcal{L}'$. We introduce further natural criteria that refined operators are expected to satisfy and we show general properties of these refined operators as well as their limits in satisfying postulates.

In fact, our main contributions are the following:

- We propose to adapt known belief revision operators to make them applicable in fragments of propositional logic. We provide natural criteria such operators should satisfy.
- Rather than restricting ourselves to the Horn fragment, we present a general framework which includes all fragments captured via closure properties on sets of models. In particular, (dual) Horn, Krom and affine formulas are thus covered.
- We characterize refined operators in a constructive way which allows us to study their properties in terms of the postulates by Katsuno and Mendelzon [3]. Most notably, we show that in case the initial operator satisfies certain postulates, then so does any of its refinements.
- We give a preliminary complexity analysis of selected refined operators. More precisely we focus on the complexity of two computational problems, namely model checking and implication. We obtain in particular completeness results for these problems within the Horn fragment for some refined Dalal and Satoh's operators.

The paper is organized as follows. After a preliminary section which introduces notations and gives a reminder on belief revision, we formally define the propositional fragments we are interested in. In Section 3 we then define refined revision operators for propositional fragments and we provide a general characterization of such refined operators that fits with any fragment. In Section 4 we study the properties of the refined operators and, in particular, show that they satisfy the basic KM postulates. Selected complexity results for some refined operators are given in Section 6. We finally report on related work in Section 7. Longer proofs appear in Appendix A.

2. Preliminaries

2.1. Propositional logic

We consider $\mathcal{L}$ as the language of propositional logic over some fixed alphabet $\mathcal{U}$ of propositional atoms. We use standard connectives $\rightarrow$, $\oplus$, $\vee$, $\wedge$, $\neg$, and constants $\top$, $\bot$. A literal is either a variable x (positive literal) or the negation $\neg x$ of one (negative literal). A clause is a disjunction of literals. A clause is called (i) *Horn* if at most one of its literals is positive; (ii) *dual Horn* if at most one of its literals is negative; (iii) *Krom* if it consists of at most two literals. A $\oplus$-clause is defined like a clause but using exclusive- instead of standard-disjunction. We identify the following subsets of $\mathcal{L}$: $\mathcal{L}_{Horn}$ as the set of all formulas in $\mathcal{L}$ being conjunctions of Horn clauses; $\mathcal{L}_{DHorn}$ as the set of all formulas in $\mathcal{L}$ being conjunctions of dual Horn clauses; $\mathcal{L}_{Krom}$ as the set of all formulas in $\mathcal{L}$ being conjunctions of Krom clauses; and $\mathcal{L}_{Affine}$ as the set of all formulas in $\mathcal{L}$ being conjunctions of $\oplus$-clauses. In what follows we sometimes just talk about arbitrary fragments $\mathcal{L}' \subseteq \mathcal{L}$. Hereby, we tacitly assume that any such fragment $\mathcal{L}' \subseteq \mathcal{L}$ contains at least the formula $\top$.

For any formula ϕ, let $\mathrm{Var}(\phi)$ denote the set of variables occurring in ϕ. An interpretation is represented either by a set $I \subseteq \mathcal{U}$ of atoms (corresponding to the variables set to true) or by its corresponding characteristic bit-vector of length $|\mathcal{U}|$. For instance if we consider $\mathcal{U} = \{x_1, \ldots, x_6\}$, the interpretation $x_1 = x_3 = x_6 = 1$ and $x_2 = x_4 = x_5 = 0$ will be represented either by $\{x_1, x_3, x_6\}$ or by $(1, 0, 1, 0, 0, 1)$. As usual, if an interpretation I satisfies a formula ϕ, we call I a model of ϕ. By $\mathrm{Mod}(\phi)$ we denote the set of all models (over $\mathcal{U}$) of ϕ. Moreover, $\psi \models \phi$ if $\mathrm{Mod}(\psi) \subseteq \mathrm{Mod}(\phi)$ and $\psi \equiv \phi$ if $\mathrm{Mod}(\psi) = \mathrm{Mod}(\phi)$.

For a set T of formulas, $Cn(T)$ denotes the closure of T under the consequence relation $\models$. A theory T is a deductively closed set of formulas $T = Cn(T)$. For fragments $\mathcal{L}' \subseteq \mathcal{L}$, we also use $T_{\mathcal{L}'}(\psi) = \{\phi \in \mathcal{L}' \mid \psi \models \phi\}$.

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