Journal of Computer and System Sciences 81 (2015) 1144–1158

Contents lists available at ScienceDirect

Journal of Computer and System Sciences

www.elsevier.com/locate/jcss

On hybrid models of quantum finite automata

Lvzhou Li [a,b,*], Yuan Feng [b,c]

[a] Department of Computer Science, Sun Yat-sen University, Guangzhou 510006, China
[b] Center for Quantum Computation and Intelligent Systems, FEIT, University of Technology Sydney, Sydney 2007, Australia
[c] AMSS-UTS Joint Research Laboratory for Quantum Computation, Chinese Academy of Sciences, Beijing, China

ARTICLE INFO

Article history:
Received 28 May 2014
Received in revised form 3 November 2014
Accepted 27 January 2015
Available online 18 February 2015

Keywords:
Quantum computing
Automata theory
Quantum finite automata
Hybrid model of QFA

ABSTRACT

In the literature, there exist several interesting hybrid models of quantum finite automata (QFA) which have both quantum and classical states. This paper describes these models in a uniform way: a hybrid QFA can be seen as a two-component communication system consisting of a quantum component and a classical one, and the existing hybrid QFA differ from each other mainly in the specific communication pattern: classical-quantum, or quantum-classical, or two-way. We clarify the relationship between these hybrid QFA and some other models; in particular, it is shown that hybrid QFA can be simulated exactly by QFA with general quantum operations. As corollaries, some results in the literature concerning the language recognition power and the equivalence problem of hybrid QFA follow directly from these relationships clarified in this paper.

1. Introduction

Quantum finite automata (QFA), as theoretical models for quantum computers with finite memory, have been explored by many researchers (see e.g. [1–7,9–19,21,22,24–27,31–33]). So far, a variety of models of QFA have been introduced and explored to various degrees (one can refer to a review article [24] and the references therein). Roughly speaking, these QFA models fall into the following two categories: one-way QFA (1QFA), where the tape head is required to move right on scanning each tape cell, and two-way QFA (2QFA), where the tape head is allowed to move left or right, and even to stay stationary. Notably, 2QFA are strictly more powerful than 1QFA: the former QFA are able to recognize[1] non-regular languages [12], while the latter QFA only regular languages [5,7,12].

Another criterion which is used to classify QFA is the state evolution type. In early references, the state evolution of a QFA was assumed to be performed by unitary operators, in accord with the postulate of quantum mechanics that the state evolution of a closed quantum system is described by a unitary transformation. Later on, it was realized that a QFA does not need to work in a closed quantum system; it can interact with an environment. Thus the state evolution should be considered as being performed by general quantum operations, i.e., trace-preserving completely positive mappings (see Hirvensalo [10,11], Li et al. [13] and Yakaryilmaz et al. [31]). QFA with general quantum operations should be thought to be a proper definition for QFA, since they possess nice closure properties and have the same computational power as their classical counterparts.

* Corresponding author at: Department of Computer Science, Sun Yat-sen University, Guangzhou 510006, China.
E-mail addresses: lilvzh@mail.sysu.edu.cn (L. Li), Yuan.Feng@uts.edu.au (Y. Feng).

[1] In this paper, recognizing a language always means recognizing it with bounded error.

http://dx.doi.org/10.1016/j.jcss.2015.01.001

Another model worth mentioning is the *ancilla QFA* proposed in [22]. Actually, ancilla QFA represent the same model as 1QFA with general quantum operations, but in different forms, which will become clear in later sections. Ancilla QFA are also closely related to *quantum sequential machines* [25,14]. The relationship between these models will be elaborated in this paper.

1.1. Hybrid models of QFA

In the literature, there is a class of QFA that differ from other QFA models by consisting of two interactive components: a quantum component and a classical one. We call them *hybrid models of QFA* in this paper. These hybrid models are of particular interest, as they show a big advantage over classical models with respect to state complexity and are easier to be physically implemented than fully quantum models (that have no classical component).

The first hybrid model of QFA was the *two-way finite automata with quantum and classical states* (2QCFA, for short) proposed by Ambainis and Watrous [4] in which on scanning an input symbol, the internal states evolve as follows: At first the quantum part undergoes a unitary operator or a projective measurement that is determined by the current classical state and the scanned symbol, and then the classical part specifies a new classical state and a movement of the tape head (left, right, or stationary), which depends on the scanned symbol (along with the outcome if the quantum part makes a measurement). In [4], it was shown that 2QCFA are strictly more powerful than their classical counterparts—two-way probabilistic finite automata (2PFA). A one-way variant of 2QCFA, denoted by 1QCFA has been described formally in [33]. It was shown in [9,32,33] that 1QCFA can be more succinct than classical (both deterministic and probabilistic) finite automata for certain languages (promise problems).

Another hybrid model considered in the literature is the *one-way quantum finite automata with control language* (CL-1QFA, for short) proposed by Bertoni [6] in which on scanning an input symbol, a unitary operator followed by a projective measurement is applied to the current quantum state. Thus, a CL-1QFA fed with an input string x finally produces a sequence y of measurement results with a certain probability. The input string x is said to be accepted if y is in a given regular language (i.e., the control language). It was shown in [6,19] that CL-1QFA recognize exactly the class of regular languages, and CL-1QFA can be more succinct (i.e., have less states) than deterministic finite automata (DFA) for certain languages.

Very recently, Qiu et al. [27] proposed a new hybrid model called *1QFA together with classical states* (1QFAC, for short) which consists of a quantum component and a classical one. On scanning an input symbol, the two components interact in the following way: At first the quantum component undergoes a unitary operator that is determined by the current classical state and the scanned symbol, and then the classical component evolves like a DFA. After scanning the whole input string, a projective measurement determined by the final classical state is performed on the final quantum state, providing the accepting and rejecting probabilities. In [27], it was proved that 1QFAC recognize all regular languages and are exponentially more concise than DFA for certain languages.

1.2. Motivation and contributions of the paper

The hybrid models mentioned above are of particular interest and worthy of considerations, at least for the following reasons. Firstly, hybrid models are easier to be physically implemented than fully quantum models. For example, while the position of the tape head in the 2QFA model introduced by Kondacs and Watrous [12] is a superposed quantum state and at least $O(\log n)$ qubits are required to store it (where n is the length of the input), in a 2QCFA the tape head position is merely a classical variable which is easy to store and manipulate. Secondly, hybrid models can often save a large number of classical states in exchange of small number of quantum states. For example, 1QCFA, CL-1QFA and 1QFAC have been proved to be much smaller than DFA when accepting some languages (resolving some promise problems). Therefore, it could be beneficial to construct a hybrid model for a practical problem (language) with an appropriate trade-off between quantum and classical states. Last but not least, quantum engineering systems developed in the future will most probably have a classical human-interactive interface and a quantum processor, and thus they will be hybrid models. In fact, hybrid models have already been encountered several times in quantum computing. For example, hybrid models appeared in the study of quantum Turing machines [29], quantum finite automata [4,6,27], multiple-prover quantum interactive proof systems [8], and quantum programming [30].[2]

This paper aims to describe these hybrid models in a uniform way and to clarify the relationship between these models and others. The contributions of this paper are as follows. (i) First, we describe three hybrid models of QFA (1QCFA, CL-1QFA, and 1QFAC) in a uniform way: each hybrid QFA is represented as a communication system consisting of a quantum component and a classical one with the communication between them encoded in controlled operations. The three models differ from each other mainly in the specific communication pattern. (ii) Second, we will show that 1QCFA, CL-1QFA, and 1QFAC can all be simulated exactly by 1QFA with general quantum operations. 1QFA with general quantum operations are actually

[2] As stated by Selinger [30], quantum programs can be described by “quantum data with classical control flows”. Thus, a quantum program can be viewed as a hybrid model where quantum data are represented by states of a quantum component and classical control flows are implemented by a classical component.