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Exponentially more concise quantum recognition of non-RMM regular languages

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ABSTRACT

We introduce a new computing model of *one-way quantum finite automata* (1QFA), namely, *one-way quantum finite automata together with classical states* (1QFAC). We show that the set of languages accepted by 1QFAC with bounded error consists precisely of all regular languages. In particular, we prove that 1QFAC are at most exponentially more concise than DFA by giving a lower bound, and we also show that this bound is tight for families of regular languages that are not recognized by measure-once (RMO), measure-many (RMM) and multi-letter 1QFA. Then, we give a polynomial-time algorithm for determining whether any two 1QFAC are equivalent. In addition, we show that the state minimization of 1QFAC is decidable within EXPSPACE.

1. Introduction

Quantum finite automata (QFA) can be thought of as a theoretical model of quantum computers in which the memory is finite and described by a finite-dimensional state space [1], as finite automata are a natural model for classical computing with finite memory [21]. As mentioned in [20], one of the motivations to study QFA is to provide some ideas to investigate the relation of classical and quantum computational complexity classes. This kind of theoretical models was firstly studied by Moore and Crutchfield [28], Kondacs and Watrous [24], and then Ambainis and Freivalds [2], Brodsky and Pippenger [13], and other authors (e.g., the references in [37]). The study of QFA is mainly divided into two ways: one is *one-way quantum finite automata* (1QFA) whose tape heads move one cell only to right at each evolution, and the other *two-way quantum finite automata* (2QFA), in which the tape heads are allowed to move towards right or left, or to be stationary. According to the measurement times in a computation, 1QFA have two types: *measure-once* 1QFA (MO-1QFA) initiated by Moore and Crutchfield [28] and *measure-many* 1QFA (MM-1QFA) studied first by Kondacs and Watrous [24]. In MO-1QFA, there is only a measurement for computing each input string, performing after reading the last symbol; in contrast, in MM-1QFA, measurement is performed after reading each symbol, instead of only the last symbol. Notably, QFA have been applied to quantum interactive proof systems [32].

MM-1QFA can accept more languages than MO-1QFA with bounded error [2], but both of them accept proper subsets of regular languages [13,11]. Another model of 1QFA with a measurement is called *multi-letter* 1QFA, proposed in [10]. In multi-letter 1QFA, there are multi-reading heads. Roughly speaking, a k-letter 1QFA is not limited to seeing only one, the just-incoming input letter, but can see several earlier received letters as well. Though multi-letter 1QFA can accept

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some regular languages not acceptable by MM-1QFA, they still accept a proper subset of regular languages. Nevertheless, as Ambainis et al. [3] mentioned, sufficient general 1QFA can indeed accept the same set of languages as DFA, for example, 1QFA *with control languages* (1QFACL, for short) proposed in [11] accept all regular languages (and only regular languages) [11,30]. However, the measurements in 1QFACL differ from those in MM-1QFA proposed in [24].

Paschen [33] presented a different 1QFA by adding some ancilla qubits to avoid the restriction of unitarity, and this model is called an ancilla 1QFA. Indeed, in ancilla 1QFA, the transition function corresponding to every input symbol is described by an isometry mapping, instead of a unitary operator. In [33], it was proved that ancilla 1QFA can recognize any regular language with certainty. With the idea in Bennett [6], Ciamarra [15] proposed another model of 1QFA whose computational power was shown to be at least equal to that of classical automata. For convenience, we call the 1QFA defined in [15] as *Ciamarra 1QFA* named after the author. In fact, the internal state of a Ciamarra 1QFA evolves by a trace-preserving quantum operation. In addition, in [27] it was proved that both ancilla 1QFA and Ciamarra 1QFA recognize only regular languages. Recently, it was proved that MO-1QFA and MM-1QFA with mixed states and trace-preserving quantum operations, instead of unitary operators, as the evolutions of states, can accept all and only regular languages [27].

These 1QFA indicated above can accept all regular languages, but their architectures are much more complicated than MO-1QFA, and more difficult to be implemented physically with present technology. Hence, proposing and exploring practical models of quantum computation is an important research problem and provide relevant insights to study physical models of quantum computers. Indeed, motivated by the implementations of quantum computers using nucleo-magnetic resonance (NMR), Ambainis et al. [1] proposed another model of 1QFA, namely, *Latvian* 1QFA (L-1QFA, for short). In L-1QFA, measurement is also allowed after reading each input symbol, but they accept a proper subset of regular languages [1]. Notably, the languages recognized with unbounded error by QFA have been discussed in [46,47].

Though ancilla 1QFA and Ciamarra 1QFA can accept all regular languages, their evolution operators of states are general quantum operations instead of unitary operators. 1QFA with pure states and unitary evolutions usually have less recognition power than *deterministic finite automata* (DFA) due to the unitarity (reversibility) of quantum physics and the finite memory of finite automata. 1QFACL can accept all regular languages but their measurement is quite complicated. However, one would expect a quantum variant to exceed (or at least to be not weaker than) the corresponding classical computing model, and such quantum computing models are practical and feasible as well. For this reason, we think that a quantum computer should inherit the characteristics of classical computers but further advance classical component by employing quantum mechanics principle.

Motivated by this idea, we propose a new model of quantum automata including a classical component, i.e., we reformulate the definition of this new model of MO-1QFA, namely, 1QFA *together with classical states* (1QFAC, for short), and in particular, we investigate some of the basic properties of this new model. As MO-1QFA [28,13], 1QFAC execute only a measurement for computing each input string, following the last symbol scanned. In this new model, we preserve the component of DFA that is used to control the choice of unitary transformation for scanning each input symbol. We now describe roughly a 1QFAC $\mathcal{A}$ computing an input string, delaying the details until Section 2.

At start up, automaton $\mathcal{A}$ is in an initial classical state and in an initial quantum state. By reading the first input symbol, the classical transformation results in a new classical state as current state, and, the initial classical state together with current input symbol assigns a unitary transformation to process the initial quantum state, leading to a new quantum state as current state. Afterwards, the machine reads the next input symbol, and similar to the above process, its classical state will be updated by reading the current input symbol and, at the same time, with the current classical state and input symbol, a new unitary transformation is assigned to execute the current quantum state. Subsequently, it continues to operate for the next step, until the last input symbol has been scanned. According to the last classical state, a measurement is assigned to perform on the final quantum state, producing a result of accepting or rejecting the input string.

Therefore, a 1QFAC performs only one measurement for computing each input string, doing so after reading the last symbol. However, the measurement is chosen according to the last classical state reached after scanning the input string. Thus, when a 1QFAC has only one classical state, it reduces to an MO-1QFA [28,13]. On the one hand, 1QFAC model develops MO-1QFA by adding DFA's component, and on the other hand, 1QFAC advance DFA by employing the fundamentals of quantum mechanics.

We want to stress that 1QFAC are not the one-way version of *two-way finite automata with quantum and classical states* (2QCFA for short) proposed by Ambainis and Watrous [4], and this version has been preliminarily considered in [49]. One of the differences is that, according to the definition of 2QCFA [4], in the one-way version of 2QCFA, after the tape head reads an input symbol, either a measurement or a unitary transformation is performed, while in 1QFAC there is no intermediate measurement, and a single measurement is performed only after scanning the input string.

Though 1QFAC make only one measurement for computing each input string and the evolutions of states are unitary instead of general operations, the set of languages accepted by 1QFAC (with no error) consists precisely of all regular languages. As we know, the set of languages accepted by 1QFACL is constituted by all regular languages [30], but 1QFACL need measurement after reading each input symbol and the measurement is not only restricted to accepting, rejecting, and non-halting, but also other results related to the control language attached to the machine. Therefore, the computing process of a 1QFACL is usually much more complicated than that of a 1QFAC. On the other hand, measuring may lead to more errors for the machine.

To prove exponential size advantages for a class of automata over DFA is a difficult problem, especially if the class of automata is such that they accept only all regular languages. Since 1QFA do not have more power than DFA in terms of