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A non-iterative linear inverse solution for the block approach in EIT

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ABSTRACT

Electrical impedance tomography (EIT) is a simple, economic and healthy technique to capture images from the internal area of the body. Although EIT is cheaper and smaller than other imaging systems and requires no ionizing radiation, the resolution associated with this technique is intrinsically limited and the image reconstruction algorithms proposed up to now are not efficient enough. In addition to low resolution EIT is an ill-posed inverse problem. Block method in EIT is based on electrical properties of materials and used to enhance image resolution and also to improve the reconstruction algorithm. Recently an inverse solution for EIT based on block method has been developed, however, this method uses non-linear algorithm. The present article provides a non-iterative linear inverse solution for the block approach on EIT. Using linear equations in this new approach provides a fast algorithm and the ability to solve complicated block problems. We have assumed that the subject has a 2D rectangular shape and is made up of identical fixed size blocks and all of the particles of each block have the same electrical conductivities. It is shown by computer simulations that this linear reconstruction algorithm employing the block method results in an accurate identification.

1. Introduction

There are a variety of medical applications for which it would be useful to know the distribution of electrical properties inside the body. By the term of "electrical properties", we mean both the electrical conductivity and permittivity, which are of interest in the medical applications [1]. Different tissues have different conductivities and permittivities, on the other hand, the knowledge of the map of the internal electrical properties has a number of advantages in the many of medical diagnosis. EIT is a useful method for medical imaging of pulmonary embolism and blood clots in the lungs [1,2], breast [3], neural system studies [4], breath system studies [5], and other medical issues.

Existing reconstruction algorithms for EIT can be categorized as following:

(1) Iterative non-linear algorithms.
(2) Iterative algorithms.
(3) Layer-stripping algorithms.
(4) Direct algorithms.

In iterative non-linear algorithms it is assumed that the conductivity differs smoothly. Examples of iterative non-linear algorithms include back-projection methods [6], Calderon's approach [7], one-step Newton methods [8], and moment methods [9]. However, in some physiological applications the conductivity variations are considerable and iterative non-linear algorithms are less useful in these cases, such as the detection of breast tumours, which are known to be two to four times more conductive than healthy breast tissues [10].

Iterative algorithms may solve the nonlinear problem. Examples of iterative algorithms include methods based on output least-squares [11,12], the equation-error formulation [13,14], or statistical inversion [15]. These algorithms are promising for obtaining accurate reconstructed values; however they often are slow on convergence.

In layer-stripping algorithms it is assumed that the higher frequencies in injected current have low penetration depth. This method is introduced and generated from inverse scattering view [16]. Convergence of this algorithm for radially symmetric problems is shown [17], however, there is no proof for its convergence in all problems and also this method is sensitive to measurement noise.

Direct algorithms represent the other class of reconstruction algorithms for EIT. They solve the nonlinear problem, so they have the potential of finding the conductivity values with high accuracy. Examples of direct algorithms based on numerical techniques may be found at Mueller's work [18] or in block method for a rectangular shape subject [19].

Block method is a new approach and in this method reconstructed conductivity values are accurate; however, equations of

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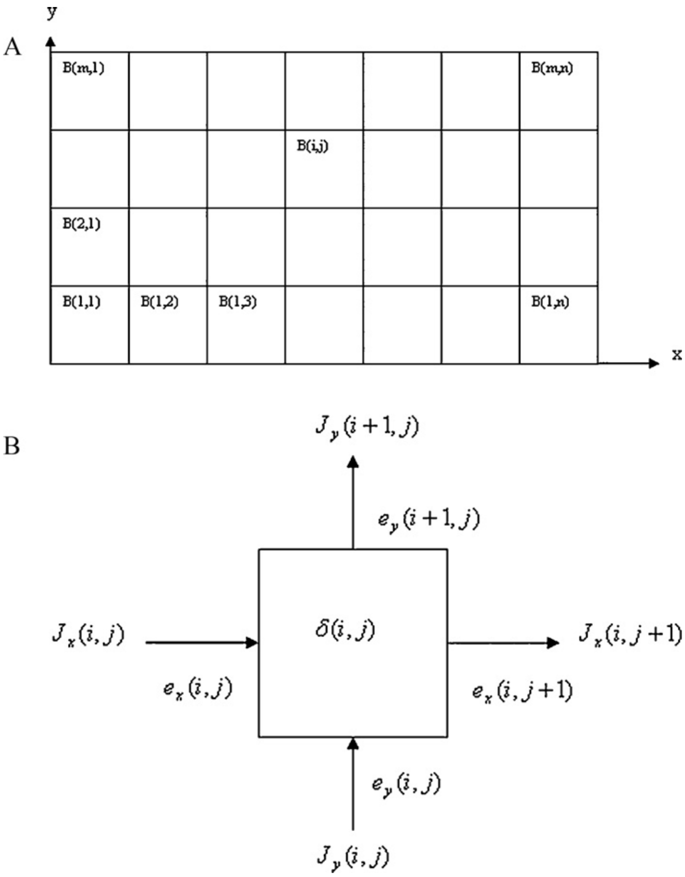

Fig. 1. (A) A schematic of a rectangular shape subject divided to $m \times n$ similar blocks. (B) Block $B(i,j)$ with $\sigma(i,j)$ specific conductivity and its current and voltage components.

reconstructed algorithm are nonlinear. The number of equations is very much and solving many nonlinear equations simultaneously encounters difficulties. In this paper, we first present the equations governing the physical problem and then describe the steps of the reconstruction algorithm in Section II. In Section III, we propose more measuring test to have a new set of data and we present linear equations for EIT based on electrical properties of material blocks. Section 4 contains a numerical example and reconstructed conductivity values from both nonlinear and linear methods. Simulation results show the validity and speed of this algorithm.

2. Block model and nonlinear reconstruction solution

To generate an EIT image, a series of electrodes are attached to a subject. Various currents can be injected through these electrodes and the produced voltages can be measured. By current injection, voltage measuring and using reconstruction algorithm, conductivity distribution inside the subject would be calculated [20,21]. EIT forward problem involves constructing a block model and calculating the voltages (or currents) produced on the boundary when currents are injected (or voltages are applied) on the same boundary [21].

In block method, a rectangular shape subject is divided into $m \times n$ similar size blocks with same electrical impedances in each block [19] as shown in Fig. 1A.

In Fig. 1A the rectangular subject has been aligned in Cartesian system and each block has been named as $B(i,j)$ (block in the ith row and jth column of Cartesian system). For a single block, $J_x(i,j)$, $J_x(i,j+1)$, $J_y(i,j)$ and $J_y(i+1,j)$ are current density components and $e_x(i,j)$, $e_x(i,j+1)$, $e_y(i,j)$ and $e_y(i+1,j)$ are voltage components for $B(i,j)$ and $\sigma(i,j)$ is specific conductivity in the whole parts of $B(i,j)$ as shown in Fig. 1B. $J_x(i,j)$ and $J_y(i,j)$ are the current densities entering the $B(i,j)$ block from X and Y directions respectively. Similarly, $e_x(i,j)$ and $e_y(i,j)$ are the voltages of the edge-centres for the $B(i,j)$ block. For $B(i,j)$ block following equations are true [19]:

$$\forall i \in N, \quad 1 \le i \le m \qquad \text{and} \qquad \forall j \in N, \quad 1 \le j \le n$$

$$e_x(i,j+1) = e_x(i,j) - \frac{1}{2}\Delta\frac{J_x(i,j)+J_x(i,j+1)}{\sigma(i,j)} \tag{1}$$

$$e_y(i+1,j) = e_y(i,j) - \frac{1}{2}\Delta\frac{J_y(i,j)+J_y(i+1,j)}{\sigma(i,j)} \tag{2}$$

$$J_x(i,j)+J_y(i,j) = J_x(i,j+1)+J_y(i+1,j) \tag{3}$$

$$\begin{aligned} e_x(i,j) - e_y(i,j) = {} & \frac{1}{8}\frac{\Delta}{\sigma(i,j)}(3J_x(i,j)+J_x(i,j+1)-3J_y(i,j) \\ & -J_y(i+1,j)) \end{aligned} \tag{4}$$

where Δ is the lengths of the block in both X and Y directions (the transverse and vertical sizes of the block).

In forward problem of EIT, $\sigma(i,j)$ are known. In this problem if the current densities on the boundaries are known the voltages on the same boundaries could be found. On the other hand if the voltages on the boundaries are known the current densities on the same boundaries could be found. The forward problem is solved as follows:

1. Getting row-number and column-number.
2. Generating $\sigma(i,j)$ for all blocks.
3. Generating $J_x(i,1)$, $J_y(1,j)$, $e_x(i,1)$ and $e_y(1,j)$.
4. Calculating $J_x(i,j)$, $J_y(i,j)$, $e_x(i,j)$ and $e_y(i,j)$ by the steps 2 and 3, using Eqs. (1)–(4).
5. Repeating steps 3 and 4 to generate enough tests.

In inverse problem of EIT, currents and voltages of boundaries are known and $\sigma(i,j)$ of blocks must be calculated. In nonlinear solution for block method for a subject with $m \times n$ blocks (Fig. 1A), in the first row from $B(1,1)$ to $B(1,n)$ we have the following equation:

$$\begin{aligned} \sigma(1,j)e_x(1,j) - 2\sigma(1,j)e_y(1,j) + \sigma(1,j)e_x(1,j+1) + \Delta J_y(1,j) \\ = 0 \quad 1 \le j \le n \end{aligned} \tag{5}$$

where $e_x(1,1)$, $J_x(1,1)$, $e_x(1,n+1)$, $J_x(1,n+1)$, $e_y(1,j)$ and $\Delta J_y(1,j)$ are known. $\sigma(1,j)$ and $e_x(1,j)$ are the unknown parameters. Eq. (5) shows n equations with $n+(n-1)=2n-1$ unknown parameters for the first row. If the measurement test is repeated by new values in the first row, different boundary currents and voltages would be resulted. Therefore new n equations with $2n-1$ unknown parameters would be obtained. It should be known that $\sigma(1,j)$ are the same in all tests, while $e_x(1,j)$ are different in each test. Therefore, each test generates n equations with $2n-1$ unknown parameters where n unknowns are common in all tests. If we try the test for n times, n^2 equations and $n+n(n-1)=n^2$ unknown parameters are obtained. n^2 unknown parameters can be solved by numerical methods and $\sigma(1,j)$ for $1 \le j \le n$ of the first row can be found.

For the second row from $B(2,1)$ to $B(2,n)$, we need the boundary values of this row of the n previously achieved tests. Boundary values $e_x(2,1)$, $J_x(2,1)$, $e_x(2,n+1)$ and $J_x(2,n+1)$ are known from the measurements. $e_y(2,j)$ and $J_y(2,j)$ are calculated by Eqs. (1)–(4) and $\sigma(2,j)$ from n^2 equations in the second row can be calculated. To find the parameters of all blocks, this procedure can be repeated for all rows.

In nonlinear solution of block method with n tests, n^2 equations are available for each row and by solving these equations the conductivities would be obtained. In next section we propose to