Journal of Computer and System Sciences 80 (2014) 1038–1045

Contents lists available at ScienceDirect

Journal of Computer and System Sciences

www.elsevier.com/locate/jcss

A complexity question in justification logic

Antonis Achilleos

The Graduate Center, CUNY, United States

ARTICLE INFO

Article history:
Received 4 January 2012
Received in revised form 16 April 2013
Accepted 3 February 2014
Available online 3 April 2014

Keywords:
Justification logic
Computational complexity
Satisfiability

ABSTRACT

Bounds for the computational complexity of major justification logics were found in papers by Buss, N. Krupski, Kuznets, and Milnikel: logics J, J4, JT, LP and JD were established to be Σ_2^p-complete. A corresponding lower bound is also known for JD4, the system that includes the consistency axiom and positive introspection. However, no upper bound has been established so far for this logic. Here, the missing upper bound for the complexity of JD4 is established through an alternating algorithm. It is shown that using Fitting models of only two worlds is adequate to describe JD4; this helps to design an effective tableau procedure and essentially is what distinguishes the new algorithm from existing ones.

1. Introduction

The classical analysis of knowledge includes the notion of justification, e.g., the famous tripartite view of knowledge as justified, true belief, usually attributed to Plato. Hintikka's modal logic approach represents knowledge as true belief. Justification logic extends epistemic logic by supplying the missing third component of Plato's characterization.

The Logic of Proofs LP was the first justification logic to be introduced, by Artemov, in [1,2] (see also [3]). Later, variations appeared in [5] corresponding to well-known normal modal logics. Several types of semantics are known for justification logics, but two are of interest in the study of complexity issues: M-models, introduced by Mkrtychev [12,17] and F-models, introduced by Fitting [4,8,9,14,18]. F-models resemble Kripke models for normal modal logics equipped with an additional mechanism, the admissible evidence function, usually denoted by $\mathcal{A}$. For a term t and formula ϕ, $\mathcal{A}(t,\phi)$ will be the set of worlds in the model where t is appropriate evidence for ϕ. $t:\phi$ is true in a world iff ϕ is true in every accessible world and the world in question is in $\mathcal{A}(t,\phi)$. M-models are essentially F-models of only one world. In this setting $\mathcal{A}(t,\phi)$ will be either *true* or *false*.

Upper and lower bounds are known for the computational complexity of justification logics J, J4, JT, LP and JD, and determine the derivability problem for these to be Π_2^p-complete [6,10,12,13,15,16]. In [15], Kuznets presents a new algorithm for checking JD-satisfiability. This algorithm is in many perspectives similar to the ones for other justification logics mentioned here: J, J4, JT, LP. A tableau method is used to try and non-deterministically construct a (description of a) model that satisfies the formula in question, then the algorithm checks whether the resulting conditions for the admissible evidence function are legitimate, thus making the product of the tableau procedure, indeed, a description of a model. This last check is known to be in coNP [10,13] and is, in fact, coNP-complete [7]. Therefore, the resulting overall algorithm is a polynomial time alternating algorithm with one alternation, starting from a universal state and eventually reaching an existential state,[1] which establishes that the problem is in Σ_2^p and therefore the logic is in Π_2^p.

E-mail address: aachilleos@gc.cuny.edu.

[1] Or, this can be viewed as an NP algorithm using an oracle from coNP.

http://dx.doi.org/10.1016/j.jcss.2014.03.009
0022-0000/

The difference among the cases that have been dealt with previously lies in the consistent evidence property of logics JD and JD4 (the "D"); in an M-model of these logics, we can never have $\mathcal{A}(t, \bot)$. In other words, if $t_1 : \phi_1, \dots, t_n : \phi_n$ are satisfied in a model, the set $\{\phi_1, \dots, \phi_n\}$ must be consistent. To incorporate this condition, the algorithm continues and tries to verify whether this set, $\{\phi_1, \dots, \phi_n\}$, is satisfiable in another model with another tableau construction. This is done by utilizing additional numerical prefixes on the prefixed formulas. A sequence of models is thus generated.

Although this construction, like all previous ones, is based on the compact character of M-models for justification logics, what it produces resembles something very similar to an F-model. In general, when discussing complexity issues for justification logics, working with F-models appears inconvenient and unnecessary: many possible worlds could succeed a current world. Furthermore, the admissible evidence function is defined on this multitude of worlds, which does not help to discuss complexity issues, especially when one is trying to confine the problem inside the Polynomial Hierarchy. On the other hand, M-models consist of only one world. The admissible evidence function is by far less complicated and the conditions it should satisfy can be checked by an NP-algorithm, except in the case of JD and JD4. For these logics, the additional condition that for any justification term t, $\mathcal{A}(t, \bot) = \mathit{false}$ cannot be verified as easily due to the negative nature of this condition. It would be nice to be able to sacrifice some, but not much, of the compact description of M-models for more convenient conditions on $\mathcal{A}$ and indeed, it seems that in the case of JD, this is exactly what can be done to provide a solution. Now, this idea will be taken a small step forward to provide a similar Σ_2 algorithm for JD4-satisfiability. Additionally, the positive introspection axiom of JD4 will help provide an even simpler class of models than in the case of JD, making the study of its complexity easier. Specifically, it is discovered that using Fitting-like models of only two worlds is adequate to describe JD4.

2. The logic JD4

JD4 was first introduced in [5] as a variation of LP, the Logic of Proofs. It is the explicit counterpart of D4, both in intuition, as there is some similarity between their axioms, and in a more precise way (see [5]).

The language will include justification constants c_i, $i \in \mathbb{N}$, justification variables: x_i, $i \in \mathbb{N}$ and justification terms, usually denoted by $t, s, \dots$. These are defined as follows.

Definition 1 *(Justification terms).*

– Constants (c_i, $i \in \mathbb{N}$) and variables (x_i, $i \in \mathbb{N}$) are terms;
– If t_1, t_2 are terms, so are

$$(t_1 \cdot t_2), \qquad (t_1 + t_2), \qquad (!t_1).$$

"$\cdot$" is usually called application, "$+$" is called sum, and "!" proof checker. The set of justification terms will be called *Tm*.

We also use propositional variables in the language: p_i, $i \in \mathbb{N}$. The set of propositional variables will be called *SLet*.

Definition 2 *(Justification formulas).* The formulas of the language are defined recursively:

If p is a propositional variable, t is a term and ϕ, ψ are formulas, then so are

$$p, \qquad \bot, \qquad (\phi \to \psi), \qquad (t : \phi).$$

$\neg\phi$ can be seen as short for $\phi \to \bot$, and the rest of the connectives can be defined from these in the usual way. As usual, parentheses will be omitted using standard conventions, and naturally, $!s : s : \phi$ will be read as $(!s : (s : \phi))$. *Fm* will denote the set of justification formulas.

The axioms of $\mathsf{JD4}_\emptyset$ are the following.

A1 Finitely many schemes of classical propositional logic;
A2 $s : (\phi \to \psi) \to (t : \phi \to (s \cdot t) : \psi)$ – Application Axiom;
A3 $s : \phi \to (s + t) : \phi$
$s : \phi \to (t + s) : \phi$ – Monotonicity Axiom;
A5 $t : \phi \to !t : t : \phi$ – Positive introspection;
A6 $t : \bot \to \bot$ – Consistency Axiom and Modus Ponens.
MP Modus Ponens Rule:

$$\frac{\phi \to \psi \quad \phi}{\psi}.$$

Definition 3. A constant specification for JD4 is any set

$$\mathcal{CS} \subseteq \{c : A \mid c \text{ is a constant, } A \text{ an axiom of JD4}\}.$$