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Realizing public announcements by justifications

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ABSTRACT

Modal public announcement logics study how beliefs change after public announcements. However, these logics cannot express the reason for a new belief. Justification logics fill this gap since they can formally represent evidence and justifications for an agent's belief. We present OPAL(K) and JPAL(K), two alternative justification counterparts of Gerbrandy–Groeneveld's public announcement logic PAL(K). We show that PAL(K) is the forgetful projection of both OPAL(K) and JPAL(K). We also establish that JPAL(K) partially realizes PAL(K). The question whether a similar result holds for OPAL(K) is still open.

1. Introduction

Justification logics are epistemic logics that feature explicit reasons for an agent's knowledge and belief. Instead of an implicit statement $\Box A$, which stands for *the agent knows A*, justification logics include explicit statements of the form $t : A$, which mean *t justifies the agent's knowledge of A*. In these statements, the evidence term t may represent a formal mathematical proof of A or an informal reason for A.

Originally, Artemov developed justification logic to provide a constructive semantics for intuitionistic logic. Later this type of logics was introduced into formal epistemology, where it provides a novel approach to several epistemic puzzles and problems of multi-agent systems [1–4,7].

Dynamic epistemic logic [9] studies the relationship between communication, knowledge, and belief. It is based on the language of modal logic enriched with statements that express various forms of communication. A basic form of communication is provided by *public announcements*, where a statement A is publicly communicated to all the agents. The logic of public announcements [18,14] uses a statement $[A]B$ to express that B holds after the public announcement of A.

In this paper, we are interested in belief rather than in knowledge and, hence, rely on Gerbrandy–Groeneveld's axiomatization of public announcements [14]. One of its postulates is

$$\Box\big(A \to [A]B\big) \leftrightarrow [A]\Box B, \tag{1}$$

which says, from left to right, that an agent who believes that B must be the case whenever a true fact A is announced will believe B after an actual announcement of A.

To illustrate how this principle works, we briefly recall the following example from [8]. If a passenger is an elite-level frequent flyer, call this statement A, this flyer can usually check in for flights at the business counter, call this statement B, provided the flyer presents the elite membership card, which can also be attached to the luggage, to make public his/her

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elite status, call this the public announcement of A. This check-in rule is known to Bob, an airline employee: $\Box_b(A \to [A]B)$. In this situation, it follows by (1) that when Ann presents her elite membership card to Bob at the business counter, he knows that he should check her in: $[A]\Box_b B$.

Modal public announcement logic tells us *how* beliefs change after public announcements but not *why*. The aim of this paper is to formalize possible answers to this *why* using the approach of justification logic. If we convert the left to right implication from (1) to a statement with explicit justifications, we should get

$$s : \big(A \to [A]B\big) \to [A]t : B, \tag{2}$$

where s represents the airline's regulations regarding business-counter check-in procedures and t is the reason why Bob starts checking Ann in.

The question is how the terms s and t, which represent justifications, relate to each other: in particular, how to arrive at t given s. We use the above example to discuss different answers to this question. There exist four possibilities:

1. $t = s$. The regulations themselves tell Bob to check Ann in. This option is implemented in the logic JPAL(K), which we developed jointly with Bryan Renne and Joshua Sack [6].
2. $t = \Uparrow s$. The operator $\Uparrow$ represents the inference Bob has to make from the regulations after the elite card is shown. This approach is taken by the logic OPAL(K), which we introduced in [8].
3. $t = \Uparrow_A s$. The inference process explicitly mentions both the regulations, s, and the demonstration of Ann's elite card, A. We do not consider this variant since it introduces an unnecessary level of complexity to the syntax: terms dependent on formulas in addition to formulas dependent on terms. Moreover, this option would go against the traditional design choices made in justification logics. For instance, the application operator $\cdot$, which corresponds to modus ponens, does not restrict how the premises of the modus ponens should look like. Similarly, the proof-checker operator !, used to verify that a given term is really a proof of a given formula, does not explicitly mention this formula because the information about all such formulas is already encoded in the term. In a similar vein, our $\Uparrow$ operator does not restrict the kind of public announcements it can be applied after and does not explicitly mention the announced formula because the information about all such formulas is already encoded in the term. Having said that, it is probably possible to develop a logic based on this third option.
4. It is also possible to consider the announcement itself to be the "physical" evidence of the announced fact. This, however, requires an update axiom different from (1) and is, in fact, incompatible with modal dynamic epistemic logic. This fourth approach is developed in [16].

As argued in [8], the simplicity of the first option, axiomatized by JPAL(K), may not always be sufficient. Imagine that Ann has been upgraded to business class (say, as a reward for postponing her original flight, which was overbooked). According to the same regulations, she can check in with Bob based on her ticket alone without announcing her elite status, which in our notation is represented by $s : B$. But Ann may choose to announce her elite status anyways, or $[A]s : B$ in our notation. In JPAL(K), where $t = s$, after the elite status is announced, t encodes two different reasons for Bob to check Ann in: as a business-class customer and as an elite flyer. By contrast, in OPAL(K), these two reasons are represented by two different terms, s and $\Uparrow s$, of which the latter depends on Ann's elite status while the former is due to the ticket alone. And Bob would want to distinguish between the two reasons because of the difference in baggage allowances: an elite frequent flyer is often allowed to check more luggage for free than the owner of a business-class ticket who has been upgraded from economy.

In addition, in this and similar cases, the approach of JPAL(K) implies that the meaning of the regulations changes after public announcements: if Ann has an economy ticket, the regulations do not allow her a business-counter check-in until she shows her elite card, and then they do. This is a little counter-intuitive since the regulations are a legal document whose meaning should not be changed by each public announcement. The use of reason $\Uparrow s$ enables us to separate the permanent status of the regulations from their momentary applications influenced by public announcements.

Let us now look at the other direction of (1)—from right to left—and see how the first two options manifest themselves there. The implication states that an agent who would believe B after an announcement of A must believe that, if A is true and announced, B would hold after the announcement. For instance, if Charlie, while standing in a line at the economy check-in counter, sees Ann being served by Bob at the business counter after she shows her elite card, $[A]\Box B$, then Charlie has empirical evidence e that Ann is served at the business counter, $[A]e : B$. It would be natural for Charlie to believe that having an elite status and showing it gets one to the business counter, $\Box(A \to [A]B)$. But it seems even clearer in this case that Charlie's empirical observation e cannot explain the causality of the implication $A \to [A]B$. If before Ann showed up, Charlie had read the sign that invited elite members to the business counter, then Charlie's memory of this sign, refreshed by Ann's actions, could serve as such an explanation. Thus, instead of using e, as in JPAL(K), in this example too it seems better to use $\Downarrow e$ as evidence for $A \to [A]B$, where $\Downarrow$ is yet another new operator of our logic OPAL(K).

So far, not much work has been done to provide explicit justifications for dynamic epistemic logic. Besides the already mentioned [6,8], research has been done on introducing new evidence [19] and eliminating unreliable evidence [21] in the framework of justification logic. Renne has also presented expressivity results for several justification logics with announcements [20]. However, the modal counterparts of Renne's systems do not correspond to any traditional public announcement