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Differential evolution for system identification of self-excited vibrations

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ABSTRACT

This study uses a differential evolution method to identify the coefficients of second-order differential equations of self-excited vibrations from a time signal. The motivation is found in the frequent occurrence of this vibration type in physics and engineering. In the proposed method, an equation structure is assumed at the level of the differential equation and a population of candidate coefficient vectors undergo evolutionary training. In this way the numerical constants of non-linear terms of various self-excited vibration types were recovered, requiring only the original signal. The method is validated by comparison with regression of the analytical solution of a linear vibration. Comparisons are given regarding accuracy and computation time. A sensitivity analysis reveals the influence of the problem stiffness and the settings of the integration algorithm for evaluating the candidate models. The algorithm is extended to allow for stability constraints based on classification of candidates by comparing with data from Monte Carlo simulations. This boosts accuracy tremendously and yields accurate coefficient values. The presented method shows promise for future applications in engineering, such as early-warning systems.

1. Introduction

This paper explores the use of evolutionary computing (EC) for the identification of self-excited vibrations, which appear in many problems in physics, biology and engineering. Examples include instabilities of train wheels, positive feedback phenomena in electrical circuits and flow-induced vibrations of gates in civil engineering structures. In the last example, interactions between the gate motion and adjacent turbulent flow can cause threatening dynamic forces on weirs and discharge sluices [12,16,10]. Because their impact is unwanted in most real-life instances, recognising and describing self-exciting oscillatory responses is of paramount concern. Apart from practical implications, they are also theoretically interesting as objects of analytical study [21].

The self-excited (or self-induced) vibration type is defined by the driving force coming from the displacement of the oscillating body itself [6]. That is, a self-sustained system exists without the need for external forcing. New energy is fed into the system through negative damping. This paper treats vibrations with one degree of freedom, which arise from a single mass-spring oscillator. The classic way to describe these mathematically is by a second-order ordinary differential equation (ODEs). The first two columns in Fig. 1 show quintessential self-excitation cases: a negative damping constant and the Van der Pol oscillator. The latter famous case has a non-linear damping term and for high enough values of the parameter μ, so-called 'relaxation vibrations' occur which contain sudden transitions with short moments of high velocity. The third example in the right column of Fig. 1 shows a non-linear mass term; to isolate this effect, an undamped oscillation is used. A standard approach is to make plots in the phase-plane; a deformed limit cycle is usually a good telltale of non-linear behaviour. In the absence of external forcing, these oscillations do not show chaotic behaviour.

The aim of our present study is to devise and test a computational method that solves the inverse problem of identifying self-excited vibrations. That is, we wish to uncover the coefficients of the ODE behind a given time series $y(t)$, which we will call the displacement signal. From the viewpoint of system identification,

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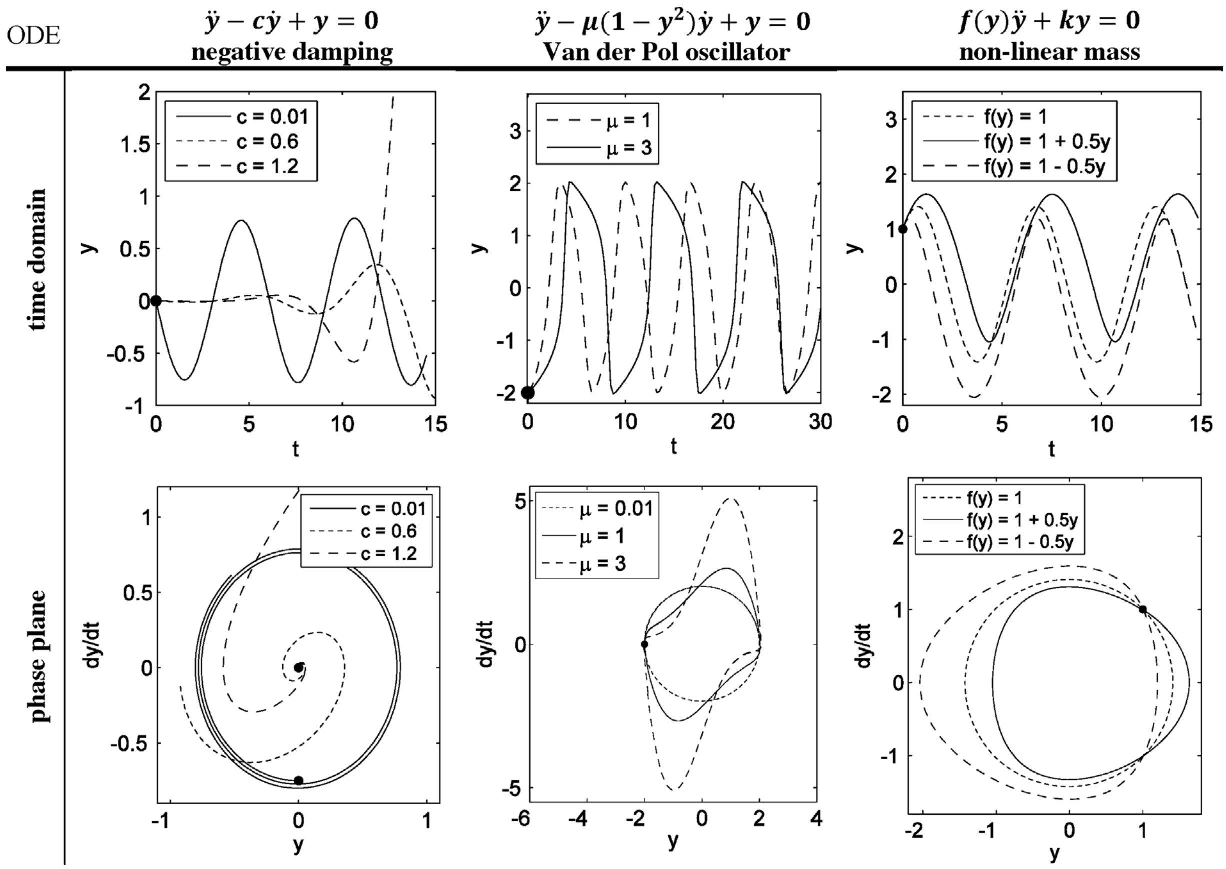

Fig. 1. Vibration examples shown in time domain (upper row) and in phase plane (lower row). Left column: constant negative damping; middle column: non-linear damping by Van der Pol oscillator; right column: non-linear mass term. The corresponding ODE equations are written on top; y is the vertical displacement of the mass, t is the independent variable time, and $\dot{y} = dy/dt$ is the first derivative. The initial states are indicated by thick dots.

we are dealing with an output-only problem where no data on the input signal (forcing) is available.

Previous work on the application of gate vibrations of hydraulic structures considered the design of a control system based on classification of sensor data and avoidance of critical levels by combining spectral analysis and machine learning [7]. The drawback of spectral analysis is that it does not detect the non-linearities that are characteristic for self-excited oscillations. The choice of a data-driven approach for this particular problem is underlined by the time-consuming efforts required to simulate the complexities and conditions in a numerical model based on fundamental physics equations [8].

System identification studies embarked on the task of inferring ODE models a few decades ago [1]. However, the fixed structure acting as a vehicle for the parameter optimization was usually not an ODE, but rather an easy-to-compute basis function like a polynomial. The advent of modern heuristics [18] and the steady increase in computing power has enormously boosted possibilities for regression of all kinds (e.g. by artificial neural networks), but many techniques do not provide clear insights into the working of the system.

A breakthrough came with the birth of genetic programming (GP) by Koza [13], which has been applied extensively to system identification problems ever since [3]. Genetic programming, part of the evolutionary algorithms family, proved to be ideal for evolving the underlying equation structures by means of symbolic regression, see e.g. [2]. Schmidt and Lipson [19] elegantly demonstrated the power of GP for identifying non-linear dynamical systems by 'discovering' physical laws and functions automatically from experimental data.

In our study, which is an extension of recent work [9], it will be assumed that the main part of the equation structure is already known, the basic second-order ODE without external forcing. The search for the remaining unknown non-linear terms could benefit from symbolic modelling, but we choose to focus on finding only the coefficients. Even apart from the extra computational burden of GP, in many practical situations a reduced number of hypotheses of equation structures exist thanks to domain expertise input. A second reason is that in GP studies the emphasis is often on the structure and the problem of determining non-trivial numerical constants is secondary, while for the application at hand this is essential. The chosen differential evolution (DE) method nevertheless facilitates a future extension of the algorithm to GP-based system identification, as the algorithmic set-up of fitness evaluation is similar.

We choose to evaluate the candidate ODE models by solving them by numerical integration. An obvious, much cheaper alternative is to differentiate the target signal twice and perform regression on the acceleration signal $\ddot{y}(t)$. But numerical differentiation is not a trivial task in this context [14]; the noise present in real-life data makes it necessary to apply some kind of filter. This is avoided in the present study, where only the original signal $y(t)$ is used and the initial value of the velocity $\dot{y}_0$. Yet other approaches use structures that are conveniently computed, such as the discrete map used by Howard and Oakley [11], but these generally provide insufficient insight in the system because the resulting expression has a format that is not easily interpreted. The idea is to really uncover the separate terms (mass, damping and stiffness) in a way that allows a meaningful analysis.