Journal of Computational Science 1 (2010) 98–106

Contents lists available at ScienceDirect

Journal of Computational Science

journal homepage: www.elsevier.com/locate/jocs

Multi-fidelity design optimization of transonic airfoils using physics-based surrogate modeling and shape-preserving response prediction

Leifur Leifsson*, Slawomir Koziel

Engineering Optimization & Modeling Center, School of Science and Engineering, Reykjavik University, Menntavegur 1, 101 Reykjavik, Iceland

ARTICLE INFO

Article history:
Received 22 March 2010
Accepted 26 March 2010

Keywords:
Aerodynamics
Airfoil shape optimization
Surrogate models
Design optimization

ABSTRACT

A computationally efficient design methodology for transonic airfoil optimization has been developed. In the optimization process, a numerically cheap physics-based low-fidelity surrogate (the transonic small-disturbance equation) is used in lieu of an accurate, but computationally expensive, high-fidelity (the compressible Euler equations) simulation model. Correction of the low-fidelity model is achieved by aligning its corresponding airfoil surface pressure distribution with that of the high-fidelity model using a shape-preserving response prediction technique. The resulting method requires only a single high-fidelity simulation per iteration of the design process. The method is applied to airfoil lift maximization in two-dimensional inviscid transonic flow, subject to constraints on shock-induced pressure drag and airfoil cross-sectional area. The results showed that more than a 90% reduction in high-fidelity function calls was achieved when compared to direct high-fidelity model optimization using a pattern-search algorithm.

1. Introduction

Aerodynamic and hydrodynamic design and optimization is of primary importance in several disciplines. In the design of turbines, such as gas, steam, or wind turbines, the blades are designed to maximize energy output for a given working fluid and operating conditions [1]. The shape of ship hull forms is optimized for minimum drag [2]. In aircraft design, for both conventional transport aircraft and unmanned air vehicles, the aerodynamic wing shape is designed to provide maximum efficiency under a variety of takeoff, cruise, maneuver, loiter, and landing conditions [3]. Constraints on aerodynamic noise are also becoming increasingly important [4]. The fundamental design problem, common to all these disciplines, is to design a wing shape that provides the desired lift for given operating conditions, while at the same time fulfilling the design constraints. However, design optimization normally requires a large number of analyses of objectives and constraints [5]. Therefore, a careful selection of computational methods, both for the fluid flow analysis and the optimization process, is essential for a fast and efficient design process.

Hicks et al. [6] began exploring the use of numerical optimization techniques for the design of aircraft components in the mid-1970s. These early studies focused primarily on airfoil and wing design at both subsonic and transonic conditions using low-fidelity flow analysis models and gradient-based optimization methods. Jameson [7] introduced control theory to optimal aerodynamic design. This method uses continuous adjoint methods to derive the gradient of a cost function with respect to the shape, and then approach the optimum using a gradient-based optimization method. Jameson and Reuther examined transonic airfoil design problems using both the full potential equation [8] and the compressible Euler equations [9]. Later, Jameson and co-workers extended the method to incorporate viscous effects using the Navier–Stokes equations, examining both two-dimensional high-lift airfoil design [10] and three-dimensional wing design [11]. Normally there is some uncertainty of the exact operating conditions, such as the Mach number and the angle of attack, therefore, much research has been on optimizing aerodynamic shapes with respect to several operating conditions [12].

The aforementioned methods directly employ the computational code in the optimization loop. In the past decade or so, the drive had been towards including higher-fidelity analyses in the design process. As a result, design optimization, which requires large numbers of model evaluations, becomes prohibitively expensive. Surrogate-based optimization (SBO) methods use computationally cheap surrogate functions in lieu of the computationally more expensive high-fidelity models [13]. The overall objective of using SBO methods is to reduce the number of evaluations of the high-fidelity models, and thereby making the optimization process more efficient. The surrogates can be created by approximating the high-fidelity model data using, e.g., polynomial regression or kriging [13]. Another way of developing the

* Corresponding author. Tel.: +354 599 6379; fax: +354 599 6201.
E-mail address: leifurth@ru.is (L. Leifsson).

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doi:10.1016/j.jocs.2010.03.007

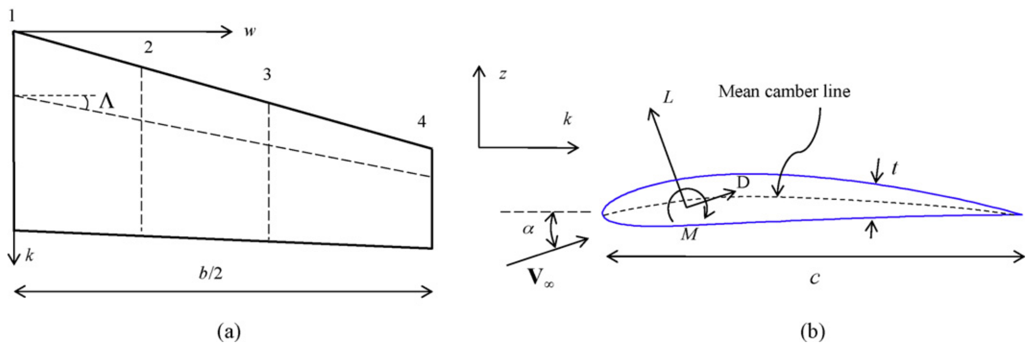

Fig. 1. (a) A schematic showing a wing planform of span b and quarter chord sweep angle Λ. Other parameters (not shown) of the wing are, e.g., taper ratio (ratio of tip chord to root chord) and twist distribution. At each spanstation (numbered 1 through 4) the wing is defined by an airfoil section with a straight line wrap between the stations; (b) Shown is a typical airfoil (NACA 2412) of chord length c, with thickness $t = t(k)$, and a mean camberline, in a free-stream with speed V_∞ at an angle of attack α. The flow generates a lift force L, a drag force D, and pitching moment M acting at the center of pressure.

surrogates is by using low-fidelity models, which are less accurate but computationally cheap representations of the high-fidelity models [14] (multi- or variable-fidelity design).

Robinson et al. [15] presented a provably convergent trust-region model-management (TRMM) methodology for variable-parameterization design models. This is an SBO method which uses a lower-fidelity model as a surrogate and the low-fidelity design space has a lower dimension than the high-fidelity design space. The mathematical relationship between the design vectors is described by a mapping method, called space mapping [16–19]. Since space mapping does not provide provable convergence within a TRMM framework, but any surrogate that is first-order accurate does, they correct the space mapping to be at least first-order, called corrected space mapping.

In summary, high-fidelity aerodynamic simulation is reliable but computationally far too expensive to be used in a direct, simulation-based design optimization, especially when using traditional, gradient-based techniques. There is a need to develop methodologies that would allow rapid design optimization with limited number of CPU-intensive objective function evaluations. SBO techniques currently used in aerospace engineering are either exploiting functional surrogate models (that require substantial computational effort to be set up), or adopt approaches such as classical space mapping (which does not guarantee sufficient alignment between the low- and high-fidelity models and requires enforced first-order consistency that required sensitivity data from the high-fidelity model). In either case, overall computational cost of the optimization process is still high. Development of a truly efficient SBO approach that would take full advantage of the low-fidelity model speed and high-fidelity model accuracy is still an open problem.

Here, a computationally efficient design methodology is introduced that exploits surrogates constructed using low-fidelity flow analysis models and shape-preserving response prediction technique [20]. We demonstrate that our approach allows a rapid design improvement of airfoils at a very low computational cost corresponding to a few evaluations of the high-fidelity model. Several examples of airfoil design at transonic flow conditions are provided.

2. Transonic airfoil aerodynamics

2.1. Airfoil geometry

A wing surface is defined by several different airfoil profiles located at several stations along the wingspan, Fig. 1(a). An airfoil profile is a streamlined surface of chord length c, as shown in Fig. 1(b). Note that here we denoted a three-dimensional Cartesian coordinate system by the variables (k,w,z). We use x and y, which are traditionally used to denote the first two coordinates, in the context of the optimization formulation (see details in Section 3).

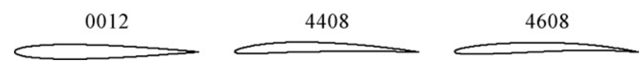

Fig. 2. Example NACA four-digit airfoils. A four-digit NACA airfoil is the simplest version of the NACA airfoil parameterization method and is defined by only three parameters m, p and t/c, where m is the maximum ordinate of the mean camber line as a fraction of chord, and p is the chordwise position of maximum ordinate. The airfoils are denoted by NACA *mpxx*, where *xx* is the thickness to chord ratio, t/c. The left most airfoil, NACA 0012, is symmetrical with no camber ($m = 0$), the location is irrelevant ($p = 0$), and 12% thickness ($t/c = 0.12$). The middle airfoil, NACA 4408, has 4% maximum camber ($m = 0.04$), located at 40% of the chord ($p = 0.4$), and 8% thickness ($t/c = 0.08$). The right most airfoil, NACA 4608, has the same camber and thickness, but the maximum camber is located at 60% of the chord ($p = 0.6$).

The airfoil has a thickness t, which is a function of k (location on the horizontal-axis), and the ratio t/c refers to the maximum thickness of the airfoil divided by its chord c. The curvature of the airfoil is called camber, and the mean camberline is the curve equidistant from the upper and lower surfaces. A few examples of airfoils parameterized according to the so-called NACA convention [21] are given in Fig. 2.

2.2. Governing equations

Transonic flow is characterized by regions of locally subsonic (Mach < 0.8) and supersonic (Mach > 1.2) flow that occurs over a body which is moving at Mach numbers near unity [22]. Assuming an inviscid, adiabatic flow with no body forces, the compressible Euler equations are the most accurate description of the fluid flow. The Euler equations are a set of coupled, non-linear partial differential equations that represent the conservation of mass, momentum and energy, i.e.,

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho V) = 0; \rho \frac{D\mathbf{V}}{Dt} = -\nabla p; \rho \frac{Dh_0}{Dt} = \frac{\partial p}{\partial t}, \tag{1}$$

where ρ is the air density, $\mathbf{V}$ is the velocity vector, p is the pressure, and h_0 is the stagnation enthalpy. These equations hold, in the absence of separation and other strong viscous effects, for any shape of the body, thick or thin, and at any angle of attack.

Shock waves appear in transonic flow where the flow goes from being supersonic to subsonic. Across the shock, there is almost a discontinuous increase in pressure, temperature, density, and entropy, but a decrease in Mach number (from supersonic to subsonic). The shock is termed weak if the change in pressure is small, and strong if the change in pressure is large. The entropy change is third order in terms of shock strength. If the shocks are weak, the entropy change across shocks is small, and the flow can be assumed to be isentropic. This in turn allows for the assumption of irrotational