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# Optimal on-line colorings for minimizing the number of ADMs in optical networks [☆]

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ABSTRACT

We consider the problem of minimizing the number of ADMs in optical networks. All previous theoretical studies of this problem dealt with the off-line case, where all the lightpaths are given in advance. In a real-life situation, the requests (lightpaths) arrive at the network on-line, and we have to assign them wavelengths so as to minimize the switching cost. This study is thus of great importance in the theory of optical networks. We present a deterministic on-line algorithm for the problem, and show its competitive ratio to be $\frac{7}{4}$. We show that this result is best possible in general. Moreover, we show that even for the ring topology network there is no on-line algorithm with competitive ratio better than $\frac{7}{4}$. We show that on path topology the competitive ratio of the algorithm is $\frac{3}{2}$. This is optimal for in this topology. The lower bound on ring topology does not hold when the ring is of bounded size. We analyze the triangle topology and show a tight bound of $\frac{5}{3}$ for it. The analyses of the upper bounds, as well as those for the lower bounds, are all using a variety of proof techniques, which are of interest by their own, and which might prove helpful in future research on the topic.



## 1. Introduction

### 1.1. Background

Optical wavelength-division multiplexing (WDM) is today the most promising technology that enables us to deal with the enormous growth of traffic in communication networks, like the Internet. A communication between a pair of nodes is done via a *lightpath*, which is assigned a certain wavelength. In graph-theoretic terms, a lightpath is a simple path in the network, with a color assigned to it.

Given a WDM network $G = (V, E)$ comprising optical nodes and a set of full-duplex lightpaths $P = \{p_1, p_2, \ldots, p_N\}$ of $G$, the wavelength assignment (WLA) task is to assign a wavelength to each lightpath $p_i$. Most of the studies in optical networks dealt with the issue of assigning colors to lightpaths, so that every two lightpaths that share an edge get different colors.

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When the various parameters comprising the switching mechanism in these networks became clearer, the focus of studies shifted, and today a large portion of the studies concentrates on the total hardware cost. The key point here is that each lightpath uses two add-drop multiplexers (ADMs), one at each endpoint. If two adjacent lightpaths, i.e. lightpaths sharing a common endpoint, are assigned the same wavelength, then they can use the same ADM. Because ADMs are designed to be used mainly in ring and path networks in which the degree of a node is at most two, an ADM may be shared by at most two lightpaths. The total cost considered is the total number of ADMs. A more detailed technical explanation can be found in [8].

Lightpaths sharing ADMs in a common endpoint can be thought as concatenated, so that they form longer paths or cycles. These paths/cycles do not use any edge $e \in E$ twice, for otherwise they cannot use the same wavelength which is a necessary condition to share ADMs.

The motivation for the on-line problem stems from the need to utilize the cost of use of the optical network. We assume that the switching equipment is installed in the network. Once a lightpath arrives, we need to assign it two ADMs, and our target is to determine which wavelength to assign to it so that we minimize the cost, measured by the total number of ADMs used.

### 1.2. Previous work

Minimizing the number of ADMs in optical networks is a main research topic in recent studies. The problem was introduced in [8] for the ring topology. An approximation algorithm for the ring topology with approximation ratio of $\frac{3}{2}$ was presented in [4], and was improved in [11,5] to $\frac{10}{7} + \epsilon$ and $\frac{10}{7}$, respectively.

For general topology [6] described an algorithm with approximation ratio of $\frac{8}{5}$. The same problem was studied in [3] and an algorithm with an approximation ratio of $\frac{3}{2} + \epsilon$ was presented. This algorithm is further analyzed in [7].

The problem of on-line path coloring is studied in earlier works, such as [10]. The problem studied in these works has a different objective function, namely the number of colors.

All previous theoretical studies on the problem of minimizing the number of switches dealt with the off-line case, where all the lightpaths are given in advance. An on-line algorithm is said to be $c$-competitive if for any sequence of lightpaths, the number of ADMs used is at most $c$ times that used by the optimal off-line algorithm (see [2]).

Recently in [1] a similar on-line scenario is considered, although in a quite different setting.

### 1.3. Our contribution

We present an on-line algorithm with competitive ratio of $\frac{7}{4}$ for any network topology. We prove that no deterministic on-line algorithm has a competitive ratio better than $\frac{7}{4}$ even if the topology is a ring.

We show that the same algorithm has a competitive ratio of $\frac{3}{2}$ in path topologies, and that this is also a lower bound for on-line algorithms in this topology.

The lower bound on ring topology does not hold when the ring is of a bounded size. We study the triangle topology, and show a tight bound of $\frac{5}{3}$ for the competitive ratio on this topology, using another algorithm.

The analyses of the upper bounds, as well as those for the lower bounds, use a variety of proof techniques, which are of interest on their own, and which might prove helpful in future research on the topic.

In Section 2 we describe the problem and some preliminary results. The algorithm and its competitive analysis are presented in Section 3. In Section 4 we present lower bounds for the competitive ratio of the problem on general topology, ring and path topologies. In Section 5 we present tight bounds for triangle networks. We conclude with discussion and open problems in Section 6.

## 2. Preliminaries

An instance $\alpha$ of the problem is a pair $\alpha = (G, P)$ where $G = (V, E)$ is an undirected graph and $P$ is a set of simple paths in $G$. In an on-line instance, the graph $G$ is known in advance and the set $P$ of paths is given on-line. In this case we denote $P = \{p_1, p_2, \ldots, p_N\}$ where $p_i$ is the $i$-th path of the input and $P_i = \{p_j \in P \mid j \leqslant i\}$ consists of the first $i$ paths of the input.

Given such an instance we define the following:

**Definition 2.1.** The paths $p, p' \in P$ are *conflicting* or *overlapping* if they have an edge in common. This is denoted as $p \asymp p'$. The graph of the relation $\asymp$ is called the conflict graph of $(G, P)$.

**Definition 2.2.** A proper coloring (or wavelength assignment) of $P$ is a function $w : P \mapsto \mathbb{N}$, such that $w(p) \neq w(p')$ whenever $p \asymp p'$.

Note that $w$ is a proper coloring if and only if for any color $c \in \mathbb{N}$, $w^{-1}(c)$ is an independent set in the conflict graph.