Journal of Computer and System Sciences 78 (2012) 960–969

Contents lists available at ScienceDirect

Journal of Computer and System Sciences

www.elsevier.com/locate/jcss

When are different type-logical semantic definitions defining equivalent meanings?

Nissim Francez

Computer Science Department, Technion-IIT, Haifa, Israel

ARTICLE INFO

Article history:
Received 9 February 2010
Received in revised form 19 May 2011
Accepted 5 August 2011
Available online 12 August 2011

To the memory of Amir Pnueli, a teacher and a friend

Keywords:
Type-logical grammar
Semantics
Types
Typed meanings
Equivalent meanings

ABSTRACT

The paper considers the issue of different type-logical semantic definitions (of some natural language fragment) being "essentially" the same, though expressed using different type-systems. A definition of the equivalence is suggested, and is applied to four definitions of a very simple extensional fragment of English.

1. Introduction

The issue dealt with in this paper is when are different model-theoretic semantic definitions in the *Type-Logical Grammar* (*TLG*) framework [5] defining equivalent semantics for (a fragment of) a natural language.

In order to allow for a more precise formulation of the question, we first recall briefly the way to define the semantics *Sem* of (a fragment $\mathcal{F}$ of) a natural language in the *TLG*, consisting of the following steps.

- Devise a *category system* $\mathcal{C}$, obtained as the closure of a finite set $\mathcal{B}$ of *basic categories* under a finite number of category-constructors. Have a distinguished (basic) category $\hat{s}$ for sentences (in the fragment). Meta-variables $\mathbf{c}, \mathbf{c}_i$ range over categories.
- Devise a *type system* $\mathcal{T}$, obtained as the closure of a finite set $\mathcal{T}_0$ of *basic types* under a finite number of type-constructors. Have a type for sentence meanings. Typically, there is a homomorphism μ from categories to types. Types are interpreted in *frames* $\mathcal{M}$, in which every basic type τ is associated with an *arbitrary* non-empty domain D_τ (possibly endowed with an algebraic structure, e.g. a boolean algebra), and type-constructors apply set operations. A *model* $\mathbf{M}$ provides a frame, and an interpretation function that assigns to each constant of type τ (see below) an element of D_τ.
- Devise a *typed terms* calculus of type inhabitants. Typically, this will be some variant of the simply-typed λ-calculus [3] (with typed constants). Refer by *sign* to a pair $\mathbf{c} : M$, where $\mathbf{c}$ is a category and M is a term of type $\mu(\mathbf{c})$. Such a sign represents a *phrase* (sequence of words) of category $\mathbf{c}$ and meaning M.
- Devise a *calculus of signs* $\mathbf{C}$, allowing reductions of *contexts* (sequences of signs $\Gamma = \mathbf{c}_1 : x_1 \cdots \mathbf{c}_n : x_n$), to a single *target sign* $\mathbf{c} : M$. The variables $\{x_1, \ldots, x_n\}$ are pairwise distinct, and are referred to as the *subjects* of Γ. This is denoted by $\vdash_{\mathbf{C}} \Gamma \rhd \mathbf{c} : M$ (a *sequent*). If the target sign is $\hat{s} : M$, it is called a *sentential* sign. Typically, the calculus is presented

E-mail address: francez@cs.technion.ac.il.

0022-0000/$ – see front matter
doi:10.1016/j.jcss.2011.08.005

as a *natural deduction* proof-system, having the term-calculus serve as *proof-terms* embodying the Curry–Howard (CH) correspondence [10].

- Devise a *lexicon* α mapping natural language (NL) words, displayed here in San-Serif font, to finite sets of *signs*. The *interpreted language* is the collection of sequences $\langle w, M\rangle$ for which there is a lexical selection of a sequence of signs, forming a context reducible in **C** to a sentential target sign. In other words, $L = \{\langle w, M\rangle \mid w = w_1 \cdots w_n,\ \Gamma = \mathbf{c}_1 : x_1 \cdots \mathbf{c}_n : x_n,\ \langle \mathbf{c}_i, M_i\rangle \in \alpha[\![w_i]\!] \text{ and } \vdash_{\mathbf{C}} \Gamma \rhd \hat{s} : M\}$. Here M is the CH proof-term associated with the derivation, having $x_1, \ldots, x_n$ as its free variables. We refer to M also as the *meaning-assembly term*. The way the *actual* meaning of w is obtained is by substituting the lexical meaning of w_i (the second component M_i of the selected element of $\alpha[\![w_i]\!]$) for x_i in M, and performing, if needed, term-reductions (typically, β-reductions). We also use the notation $[\![w]\!]^{Sem} = M$ (omitting *Sem* when clear from context). Note that while meaning-assembly terms are *pure*, in that they are formed using typed variables only, lexical meanings typically use typed *constants*, interpreted by the model as elements in the type of their domain (like predicate symbols in first-order logic (FOL)). We adhere here to the convention that such constants are displayed in boldface font.
  Furthermore, for every sentence φ (of category $\hat{s}$), there is a given notion of *satisfaction* of φ in models in **M**, relative to a variable assignment v (for free variables), denoted by $\mathbf{M}, v \models \varphi$. As usual, $\mathbf{M} \models \varphi$ iff $\mathbf{M}, v \models \varphi$ for every v.

We now can ask the question more precisely. Assuming we fixed the words in the fragment, when should two different definitions, possibly using different categories, collections of basic types and type-constructors, considered to be "really" defining the same semantics? The considerations for choosing a collection of basic types and type-constructors may vary from ontological commitments to efficiency of computation, and are not discussed here. We just want to study their effect on the overall definition.

We propose an answer to this question, and examine it on several exemplary minimally different semantic definitions of a very small fragment of English, using different basic categories and types, as well as different type-constructors, and see what is involved in realizing they really capture, in a sense, the same semantic definition of the fragment. The main difference between the various definitions presented is in the way *predication* and *quantification* are viewed. We start by considering definitions varying in basing types, but all having *functional types* as their compound types. Then, we also examine a definition based on *relational types*.

2. Relating different meaning definitions

Suppose a fragment $\mathcal{F}$ has been fixed. Consider two semantic definitions of the fragment, to be referred to as Sem_1, Sem_2, respectively, using different type-signatures, say $\mathcal{T}_1, \mathcal{T}_2$, respectively. Let $\mathcal{M}_1, \mathcal{M}_2$ be frames for the two type-signatures.

Two models $\mathbf{M}_1, \mathbf{M}_2$ (over $\mathcal{M}_1, \mathcal{M}_2$, respectively) are *$\mathcal{F}$-compatible*, denoted by $\mathcal{R}_{\mathcal{F}}(\mathbf{M}_1, \mathbf{M}_2)$, iff for every φ of category $\hat{s}$,

$$\mathbf{M}_1 \models \varphi \quad \text{iff} \quad \mathbf{M}_2 \models \varphi$$

Such a pair of $\mathcal{F}$-compatible models specifies the same informal state of affairs, by satisfying the same subset of $\mathcal{F}$-sentences, each representing this state of affairs by means of its type-signature.

We say that the two given semantic definitions are $\mathcal{F}$-equivalent iff the following two conditions hold:

1. For every $\mathbf{M}_1$ (over $\mathcal{M}_1$), there exists some $\mathbf{M}_2$ (over $\mathcal{M}_2$) s.t. $\mathcal{R}_{\mathcal{F}}(\mathbf{M}_1, \mathbf{M}_2)$.
2. For every $\mathbf{M}_2$ (over $\mathcal{M}_2$), there exists some $\mathbf{M}_1$ (over $\mathcal{M}_1$) s.t. $\mathcal{R}_{\mathcal{F}}(\mathbf{M}_1, \mathbf{M}_2)$.

Thus, for equivalent semantic definition, for each way of specifying some state of affair by means of a model based on one type-signature, there is a model using the other type-signature, specifying the same state of affairs. It is in this sense that the two equivalent semantic definitions define the fragment "in the same way".

3. Preliminaries

3.1. The fragment

We consider an *extensional* fragment E_0^- of English, comprising the following words.

- *proper names*: Rachel, Jacob, ...
- *Nouns*: girl, shepherd, ... (only singular, count nouns).
- *Intransitive verbs*: smile, dream, ...

The above word classes are *open* classes, having the property that their meaning is *any* element in the domain of interpretation of their type, hence mutually indistinguishable within each class. Thus, it suffices to include a single representative of each open class in the actual grammars considered. There is also a *closed* class of words, the members of which having a unique semantic contribution.

- *Determiners*: Every, some, no, the, ...