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Bothway embedding of circulant network into grid ☆

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ABSTRACT

Graph embedding is an important technique that maps a guest graph into a host graph, usually an interconnection network. In this paper, we compute the dilation and wirelength of embedding circulant network into grid and vice versa.

1. Introduction

A parallel algorithm or a massively parallel computer can each be modeled by a graph, in which the vertices of the graph represent the processes or processing elements, and the edges represent the communications among processes or processors. Thus, the problem of efficiently executing a parallel algorithm A on a parallel computer M can be often reduced to the problem of mapping the graph G, representing A, on the graph H, representing M, so that the mapping satisfies some predefined constraints. This is called a graph embedding [3].

An embedding of a guest graph G into a host graph H is a one-to-one mapping of the vertex set of G into that of H. The quality of an embedding can be measured by certain cost criteria. One of these criteria which is considered very often is the *dilation*. The dilation of an embedding is defined as the maximum distance between pairs of vertices of H that are images of adjacent vertices of G. It is a measure for the communication time needed when simulating one network on another [12].

Another important cost criterion is the *wirelength*. The wirelength of a graph embedding arises from VLSI designs, data structures and data representations, networks for parallel computer systems, biological models that deal with cloning and visual stimuli, parallel architecture, structural engineering and so on [22,39].

The circulant graph is a natural generalization of the double loop network [38] and has been used for decades in the design of computer and telecommunication networks due to its optimal fault-tolerance and routing capabilities [7]. It is also used in VLSI design and distributed computation [1,2,37]. Circulant graphs have been employed for designing binary codes [20]. Theoretical properties of circulant graphs have been studied extensively and surveyed by Bermond et al. [2]. Every circulant graph is a vertex transitive graph and a Cayley graph [39]. Most of the earlier research concentrated on using the circulant graphs to build interconnection networks for distributed and parallel systems [2,7].

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Graph embeddings have been well studied for hypercubes into grids [24], meshes into crossed cubes [14], meshes into locally twisted cubes [17], meshes into faulty crossed cubes [40], generalized ladders into hypercubes [8], rectangular grids into hypercubes [10], rectangular grids into hypercubes [13], grids into grids [34], binary trees into grids [28], meshes into Möbius cubes [36], circulant into arbitrary trees, cycles, certain multicyclic graphs and ladders [33], tori and grids into twisted cubes [21].

Even though there are numerous results and discussions on the wirelength problem, most of them deal with only approximate results and the estimation of lower bounds [3,9]. But the Congestion Lemma and the Partition Lemma [24] have enabled to obtain exact wirelength of embeddings for various architectures [23–25,29,31–33]. This technique focuses on specific partitioning of the edge set of the host graph. It is interesting to note that not all host graphs can be partitioned to apply the Partition Lemma. In this paper, we overcome this difficulty by partially retaining a set of edges on which the wirelength is computed using Partition Lemma and compute minimum congestion on the rest of the edges using certain other procedure.

The paper is organized as follows. Section 2 gives definitions and other preliminaries. Sections 3 and 4 establish the main results. Finally, concluding remarks and future work are given in Section 5.

2. Basic concepts

In this section we give the basic definitions and preliminaries related to embedding problems.

Definition 2.1. (See [3].) Let G and H be finite graphs. An *embedding* $\phi = (f, P_f)$ of G into H is defined as follows:

1. f is a one-to-one map from $V(G) \to V(H)$.
2. P_f is a one-to-one map from $E(G)$ to $\{P_f(u,v) : P_f(u,v)$ is a path in H between $f(u)$ and $f(v)$ for $(u,v) \in E(G)\}$.

For brevity, we denote the pair (f, P_f) as f.

Definition 2.2. (See [3].) If $e = (u,v) \in E(G)$, then the length of $P_f(u,v)$ in H is called the *dilation* of the edge e. The maximal dilation over all edges of G is called the dilation of the embedding f. The dilation of embedding G into H denoted by $d(G,H)$ is the minimum dilation taken over all embeddings f of G into H. The *expansion* of an embedding f is the ratio of the number of vertices of H to the number of vertices of G.

In this paper, we consider embeddings with expansion one.

The *edge congestion* of an embedding f of G into H is the maximum number of edges of the graph G that are embedded on any single edge of H. Let $EC_f(e)$ denote the number of edges (u,v) of G such that e is in the path $P_f(u,v)$ between $f(u)$ and $f(v)$ in H.

In other words,

$$EC_f(e) = \left|\left\{(u,v) \in E(G) : e \in P_f(u,v)\right\}\right|$$

where $P_f(u,v)$ denotes the path between $f(u)$ and $f(v)$ in H with respect to f. On the other hand, if S is any subset of $E(H)$, then $EC_f(S) = \sum_{e \in S} EC_f(e)$.

If we think of G as representing the wiring diagram of an electronic circuit, with the vertices representing components and the edges representing wires connecting them, then the edge congestion $EC(G,H)$ is the minimum, over all embeddings $f : V(G) \to V(H)$, of the maximum number of wires that cross any edge of H [4]. See Fig. 1.

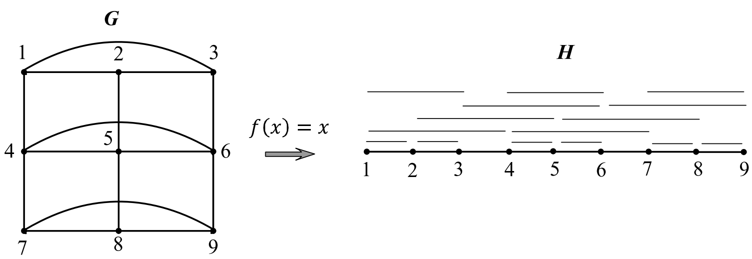

Fig. 1. Wiring diagram of a cylinder G into path H with $WL_f(G,H) = 30$.