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Computing with membranes and picture arrays

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ABSTRACT

Splicing systems were introduced by Tom Head [3,4] on biological considerations to model certain recombinant behaviour of DNA molecules. An effective extension of this operation to images was introduced by Helen Chandra et al. [5] and *H* array splicing systems were considered. A new method of applying the splicing operation on images of hexagonal arrays was introduced by Thomas et al. [12] and generated a new class of hexagonal array languages HASSL. On the other hand, *P* systems, introduced by Paun [6] generating rectangular arrays and hexagonal arrays have been studied in the literature, bringing together the two areas of theoretical computer science namely membrane computing and picture languages. *P* system with array objects and parallel splicing operation on arrays is introduced as a simple and effective extension of *P* system with operation of splicing on strings and this new class of array languages is compared with the existing families of array languages. Also we propose another *P* system with hexagonal array objects and parallel splicing operation on hexagonal arrays is introduced and this new class of hexagonal array languages is compared with the existing families of hexagonal array languages.

1. Introduction

Models based on biological phenomena that were introduced in the literature enriched both formal language theory and life science with major developments. Splicing system is one such model introduced by Head [3] based on biological considerations. The splicing systems make use of a new operation, called splicing on strings of symbols. Helen Chandra et al. [5] extended this operation and introduced a new method of splicing on images of rectangular arrays.

On the other hand, *P* systems introduced by Paun [6] are a class of distributed parallel computing devices of biochemical inspiration. These systems are based on a structure of finitely many cell membranes which are hierarchically arranged. All cell membranes are embedded in a main membrane called skin membrane. The membranes delimit regions where objects, elements of a finite alphabet, and evolution rules present. Evolution rules may contain target indicators; here indicates that the resulting object remains in the same membrane where it is produced; out indicates that the resulting object is sent to the region surrounding the membrane in which it is produced; in indicates that the resulting object is sent to a membrane which is contained in that membrane. Many variants of *P* systems have been introduced and extensively studied which can be seen in [7].

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Also, in the study of picture generation, several grammars were introduced in the literature to generate various classes of pictures. One such grammar is the hexagonal kolam array grammar (HKAG) introduced by Siromoney and Siromoney [10], generating a class of hexagonal arrays (HKAL). A new method of applying the parallel splicing operation on images of hexagonal arrays that involve 2×1 or 1×2 dominoes along x or y directions are introduced in [12].

In this paper, we introduce a new P system called array splicing P system where the objects are rectangular arrays and the evolution rules are parallel splicing rules as introduced in [5]. We compare the family of languages generated by this system with existing families of array languages like local two-dimensional array languages. Also we propose another P system called parallel splicing Hexagonal Array P System where the objects are hexagonal arrays and the evolution rules are parallel splicing rules as introduced in [12]. We compare the family of hexagonal array languages generated by this system with existing families of hexagonal array languages like hexagonal local picture languages.

2. Basic definitions

In this section, we recall the basic definition of array languages, hexagonal picture languages and parallel splicing rules over array and hexagonal pictures as in [5,12]. For the basic definition of P system and its variants, we refer to [6,7].

Definition 2.1. Let V be a finite alphabet. A picture A over V is a rectangular $m \times n$ array of elements of the form

$$A = \begin{matrix} a_{11} & \dots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{m1} & \dots & a_{mn} \end{matrix} = [a_{ij}]_{m \times n}$$

The set of all pictures or arrays over V is denoted by V^{**}. A picture or an array language over V is a subset of V^{**}.

Definition 2.2. Let V be an alphabet, # and \$ are two symbols that are not in V. A vertical domino is of the form $\begin{array}{|c|}\hline a \\ \hline b \\ \hline\end{array}$, and a row domino is of the form $\begin{array}{|c|c|}\hline a & b \\ \hline\end{array}$, where $a, b \in V \cup \{\lambda\}$.

A domino column splicing rule over V is of the form $p : y_1 \# y_2 \$ y_3 \# y_4$, where $y_i = \begin{array}{|c|}\hline a \\ \hline b \\ \hline\end{array}$ or $\begin{array}{|c|}\hline \lambda \\ \hline \lambda \\ \hline\end{array}$, $1 \le i \le 4$.

A domino row splicing rule over V is of the form $q : x_1 \# x_2 \$ x_3 \# x_4$, where $x_i = \begin{array}{|c|c|}\hline a & b \\ \hline\end{array}$ or $x_i = \begin{array}{|c|c|}\hline \lambda & \lambda \\ \hline\end{array}$, $1 \le i \le 4$.

Let

$$X = \begin{matrix} a_{11} & \dots & a_{1,j} & a_{1,j+1} & \dots & a_{1p} \\ a_{21} & \dots & a_{2,j} & a_{2,j+1} & \dots & a_{2p} \\ \dots & \dots & \dots & \dots & \dots & \dots \\ a_{m1} & \dots & a_{m,j} & a_{m,j+1} & \dots & a_{mp} \end{matrix}$$

and

$$Y = \begin{matrix} b_{11} & \dots & b_{1,k} & b_{1,k+1} & \dots & b_{1q} \\ b_{21} & \dots & b_{2,k} & b_{2,k+1} & \dots & b_{2q} \\ \dots & \dots & \dots & \dots & \dots & \dots \\ b_{m1} & \dots & b_{m,k} & b_{m,k+1} & \dots & b_{mq} \end{matrix}$$

We write $(X, Y) \vdash_c Z$ if there exist column splicing rules $p_1, p_2, \dots, p_{m-1}$, not all necessarily different, such that

$$p_i = \begin{array}{|c|}\hline a_{i,j} \\ \hline a_{i+1,j} \\ \hline\end{array} \# \begin{array}{|c|}\hline a_{i,j+1} \\ \hline a_{i+1,j+1} \\ \hline\end{array} \$ \begin{array}{|c|}\hline b_{i,k} \\ \hline b_{i+1,k} \\ \hline\end{array} \# \begin{array}{|c|}\hline b_{i,k+1} \\ \hline b_{i+1,k+1} \\ \hline\end{array}$$

for all i $(1 \le i \le m-1)$ and for some j, k $(1 \le j \le p-1,\ 1 \le k \le q-1)$, and

$$Z = \begin{matrix} a_{11} & \dots & a_{1,j} & b_{1,k+1} & \dots & b_{1q} \\ a_{21} & \dots & a_{2,j} & b_{2,k+1} & \dots & b_{2q} \\ \dots & \dots & \dots & \dots & \dots & \dots \\ a_{m1} & \dots & a_{m,j} & b_{m,k+1} & \dots & b_{mq} \end{matrix}$$

In a similar way, row splicing operation of two images U and V of sizes $p \times n$ and $q \times n$ using row splicing rules will produce an image W as given below: