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Excessive index for mesh derived networks

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ABSTRACT

A matching in a graph $G = (V, E)$ is a subset M of edges, no two of which have a vertex in common. A matching M is said to be perfect if every vertex in G is an endpoint of one of the edges in M. The excessive index of a graph G is the minimum number of perfect matchings to cover the edge set of G. In this paper we determine the excessive index for mesh, cylinder and torus networks.

1. Introduction

The interconnection network is responsible for fast and reliable communication among the processing nodes in any parallel computer [11]. Processing and distribution of data using interconnection networks have become indissoluble elements of the development of our society. Many systems consider communications among internal entities as a key factor in their performance. Examples of these systems are VLSI (very large-scale integration) circuits, image processing, simulations of diverse types of chemical reactions, telephone networks, computer networks and many others. The need for ever increasing computing power is a current problem in modern technology. Parallel computing with multiple processors is a feasible approach to tackle this problem. To implement this approach many communication schemes are necessary, including the interconnection of processors. Thus network design concepts become imperative elements in our life.

Various research and development results on how to interconnect multiprocessor components have been reported in literature. One of the most popular architectures is the mesh-connected computer, in which processors are placed in a square or rectangular grid, with each processor being connected by a communication link to its neighbors in up to four directions. Tori are meshes with wrap around connections to achieve vertex and edge symmetry. Meshes and tori are among the most frequent multiprocessor networks available today in the market [16].

A classic problem in graph theory and theoretical computer science is that of finding subgraphs of a given graph with prescribed vertex degrees. Subgraphs of prescribed vertex degrees are commonly referred to as factors [8]. A k-factor of graph G is defined as a k-regular spanning subgraph of G. A *matching* in a graph $G = (V, E)$ is a subset M of edges, no two of which have a vertex in common. A matching M is said to be *perfect* (or 1-factor) if every vertex in G is an endpoint of one of the edges in M. A perfect matching of a graph is a spanning subgraph which is regular of degree one. A *near-perfect* (or near 1-factor) matching covers all but exactly one vertex. Tutte has characterized graphs which contain

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1570-8667/

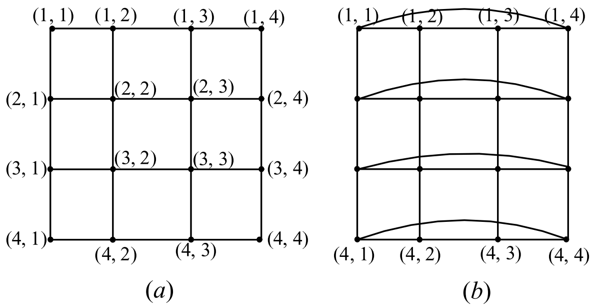

Fig. 1. (a) 4×4 mesh network connecting 16 nodes (b) 4×4 cylinder network connecting 16 nodes.

1-factors [18]. Beineke and Plummer [1] proved that every block with a 1-factor always contains at least one more, and a result due to Petersen [14] showed that every cubic graph with no bridges contains a 1-factor [17]. There are a number of famous conjectures and open problems on perfect matchings including Berge–Fulkerson conjecture, Fan Raspaud conjecture and problems on maximum or minimum number of perfect matching, induced matching partition, matching preclusion and many others including *excessive index* problem.

2. An overview of the paper

A graph G is *1-extendable* if every edge of G belongs to at least one 1-factor of G. A 1-*factor* cover of G is a set F of 1-factors of G such that $\bigcup_{F \in \mathcal{F}} F = E(G)$. A 1-factor cover of minimum cardinality is called an *excessive factorization* [3]. The *excessive index* of G, denoted $\chi'_e(G)$, is the size of an excessive factorization of G. We define $\chi'_e(G) = \infty$ if G is not 1-extendable. A graph G is 1-factorizable if its edge set $E(G)$ can be partitioned into edge-disjoint 1-factors. An *excessive near* 1-*factorization* of a graph G is a minimum set of near 1-factors whose union contains all the edges of G [5]. Excessive index has a number of applications particularly in *scheduling theory* to complete the process in minimum possible time [6]. The problem of determining whether a regular graph G is 1-factorizable is *NP*-complete [10].

Bonisoli et al. [3] observed that the problem of determining the excessive index for regular graphs is *NP*-hard. Cariolaro et al. [4] determined the excessive index of complete multipartite graphs, which proved to be a challenging task. The excessive index of a bridgeless cubic graph has been studied by Fouquet et al. [9]. Further excessive index are being calculated for regular graphs in [2,13]. Rajasingh et al. have determined excessive index for honeycomb [15], butterfly [15], hexagonal [12] and 3-D mesh network [12]. In general, it is proved that $\chi'_e(G) \geq \chi'(G)$ where $\chi'(G)$ is the edge-chromatic number (chromatic index) of G and that the difference between $\chi'_e(G)$ and $\chi'(G)$ can be arbitrarily large [3]. In this paper we determine the excessive index for mesh, cylinder and torus networks.

3. A general results on excessive index

Theorem 1. *(See [3].) Let G be a graph. Then $\chi'_e(G) \geq \Delta$.*

Theorem 2. *(See [7].) Every r-regular bipartite graph, $r \geq 1$, is 1-factorable.*

Theorem 3. *Let G be a regular bipartite graph of even order. Then $\chi'_e(G) = \Delta$.*

Theorem 4. *Let $G(V, E)$ be a graph of odd order with maximum degree Δ. If $|E| > \Delta \times \lfloor \frac{|V(G)|}{2} \rfloor$, then $\chi'_e(G) \geq \Delta + 1$.*

Proof. Each of the perfect matchings in G covers $\lfloor \frac{|V(G)|}{2} \rfloor$ edges. Thus $\bigcup_{1 \leq i \leq \Delta} M_i$ will cover at most $\Delta \times \lfloor \frac{|V(G)|}{2} \rfloor$ edges. □

4. Excessive index of mesh derived networks

Definition 1. Let P_n denote a path on n vertices. For $m, n \geq 2$, $P_m \times P_n$ is defined as the two dimensional mesh with m rows and n columns. It is denoted by $M_{m \times n}$. See Fig. 1(a).

Definition 2. Let C_n and P_n denote a cycle and a path on n vertices respectively. For $m, n \geq 2$, $C_m \times P_n$ is defined as the two dimensional cylinder with m rows and n columns. It is denoted by $CY_{m \times n}$. See Fig. 1(b).

Definition 3. Let C_n denote a cycle on n vertices. For $m, n \geq 2$, $C_m \times C_n$ is defined as the two dimensional torus with m rows and n columns. It is denoted by $T_{m \times n}$. See Fig. 2.