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A new view on Rural Postman based on Eulerian Extension and Matching ☆

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ABSTRACT

We provide a new characterization of the NP-hard arc routing problem Rural Postman in terms of a constrained variant of minimum-weight perfect matching on bipartite graphs. To this end, we employ a parameterized equivalence between Rural Postman and Eulerian Extension, a natural arc addition problem in directed multigraphs. We indicate the NP-hardness of the introduced matching problem. In particular, we use the matching problem to make partial progress towards answering the open question about the parameterized complexity of Rural Postman with respect to the parameter "number of weakly connected components in the graph induced by the required arcs". This is a more than thirty years open and long-neglected question with significant practical relevance.

1. Introduction

The Rural Postman (RP) problem [13,28] with its special case, the Chinese Postman problem [26], is a famous arc routing problem in combinatorial optimization. Given a directed, arc-weighted graph G and a subset R of its arcs (called "required arcs"), the task is to find a minimum-cost closed walk in G that visits all arcs of R. The practical applications of RP include snow plowing, garbage collection, and mail delivery [1,3,5,12,14,32]. Recently, it has been observed that RP is closely related (more precisely, "parameterized equivalent") to the arc addition problem Eulerian Extension (EE) [10].

In EE, a directed multigraph G and a function assigning a weight value to each potential arc on the vertices of G are given. The task is to find a minimum-weight set of arcs to add to G such that the resulting multigraph is Eulerian. RP and EE are NP-hard [23,22]. In fact, their mentioned parameterized equivalence means that many algorithmic and complexity-theoretic results for one of them transfer to the other. In particular, this gives a new view on RP, perhaps leading to novel approaches to attack its computational hardness.

A key issue in both problems is to determine the influence of the number c of connected components on each problem's computational complexity [10,18,17,23,29]. More precisely, c refers to the number of weakly connected components in the input graph for EE and the number of weakly connected components in the graph induced by the required arcs for RP.

☆ This work is based on the Diploma thesis of one of the authors (Sorge, 2011 [33]). A preliminary version of this work has been presented at the 22nd International Workshop on Combinatorial Algorithms (IWOCA'11), Victoria, Canada, June 2011 (Sorge et al., 2011 [35]). We also give full versions of some results which have been presented at the 37th International Workshop on Graph-Theoretic Concepts in Computer Science (WG'11), Teplá Monastery, Czech Republic, June 2011 (Sorge et al., 2011 [34]).

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If $c = 1$, then RP is efficiently solvable in polynomial-time [10]. Indeed, Frederickson [18,17] observed that, generally, RP is polynomial-time solvable when c is constant. However, c influences the degree of the polynomial in the running time of Frederickson's algorithm. To date, it is open whether this is unavoidable[4] or whether RP can be solved in $f(c) \cdot n^{O(1)}$ time for some function f. In other words, it remains open whether RP (and EE) is fixed-parameter tractable with respect to the parameter c [10]. See Section 2 and the literature [11,15,27] for more on parameterized complexity analysis. We remark that the parameter c is presumably small in a number of applications [10,18,17]. This motivates addressing this seemingly hard open question.

1.1. Related work

The RP problem and its various variants have received much attention in the past. Subsequent to RP's introduction [13,28] it has been shown NP-complete [23]. Heuristics and approximation algorithms have been presented [18,17,30,3,20] as well as exact exponential-time algorithms based on integer linear programs [7,8,19,25]. See also overview articles by Eiselt et al. [14], by Assad and Golden [1] and the book edited by Dror [12]. There is also a number of papers that evaluate algorithms for RP in practical settings [5,31]. However, we are not aware of studies in the realm of parameterized complexity except in the context of Eulerian extensions.

Höhn et al. [22] recently introduced a variant of EE in the context of scheduling and proved it to be NP-complete. EE has been shown to be polynomial-time solvable in some special cases [4,24,10,22]. Dorn et al. [10] also proved that EE is fixed-parameter tractable with respect to the parameter "number of arcs in the sought Eulerian extension". Note that this parameter is an upper bound for c, however, it is reasonable to assume that c is much smaller in practice. Also, the parameterized complexity of a number of vertex and edge deletion problems related to Eulerian graphs has been considered recently [6,9,16].

1.2. Our results

In this work, we contribute new insights concerning the seemingly hard open question whether RP (and EE) is fixed-parameter tractable with respect to the parameter "number c of components". To this end, our main contribution is a new characterization of RP in terms of a variant of minimum-weight perfect matching on (undirected) bipartite graphs: Conjoining Bipartite Matching (CBM). Here, in addition to searching a matching that matches every vertex and that is of weight at most some given maximum, further constraints are given: The vertices in the input graph are grouped and the additional constraints are of the form "between vertex group A and vertex group B, there must be at least one edge in the matching". A more formal definition is given in Section 4. We show that EE and CBM are parameterized equivalent with respect to the parameters "number of components" for EE and "number of additional constraints" for CBM.

To prove the equivalence of EE and CBM, we use a *parameterized Turing reduction*; thus, we have to separately show that CBM is still NP-hard under classical many-one reductions. As it turns out, this is the case even when the input graph has maximum degree two. We address the open question of whether EE is fixed-parameter tractable with respect to the parameter "number of weakly connected components": We obtain that CBM is fixed-parameter tractable with respect to the parameter "number of additional constraints" when restricted to bipartite graphs where one partition set has maximum vertex degree two. This implies corresponding fixed-parameter tractability results for relevant special cases of RP and EE which would perhaps have been harder to formulate and to detect using the original definitions of these problems. Indeed, we hope that CBM might help to finally answer the puzzling open question concerning the parameterized complexity of RP with respect to the number c of components.

As a side result, we also obtain a fixed-parameter algorithm for EE from one of the reductions we give. It implies that EE is fixed-parameter tractable with respect to the parameters c and "the sum b of positive balances of vertices in the input". Together, these parameters measure the problem's distance from triviality [21].

In this paper, we focus on decision problems. However, our results easily transfer to the corresponding optimization problems. Note that, for the sake of notational convenience and justified by the known parameterized equivalence [10], most of our results and proofs refer to EE instead of RP.

1.3. Structure of the paper

This work is organized as follows. In Section 2, we provide some notation, preliminary observations and useful results. Next, the parameterized equivalence of RP and CBM is proven in two steps. First, in Section 3, variants of EE are introduced and reductions are given that are used as intermediate steps for the reductions that yield the equivalence. This also yields the above-mentioned fixed-parameter algorithm for EE with respect to the parameters b and c. Second, in Section 4, it is shown that CBM can be reduced to one of the variants of EE and another variant of EE can be reduced to CBM. This then concludes the proof of equivalence of CBM, EE, and, thus, RP. Next, in Section 5, we take a closer look at CBM. In particular, we show the fixed-parameter tractability for the mentioned special case. See Fig. 1 for an overview of the reductions given in the paper. We conclude in Section 6 with directions for future research.

[4] Under reasonable complexity-theoretic assumptions.