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Routing permutations and involutions on optical ring networks: complexity results and solution to an open problem

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Abstract

Given a network G and a demand D of communication requests on G, a routing for (G, D) is a set of directed paths of G, each from the source to the destination of one request of D. The ROUTING AND WAVELENGTH ASSIGNMENT PROBLEM seeks a routing $\mathcal{R}$ for (G, D) and an assignment of wavelengths to the directed paths in $\mathcal{R}$ such that the number of wavelengths used is minimized, subject to that any two directed paths with at least one common arc receive distinct wavelengths. In the case where G is a ring, this problem is known as the RING ROUTING AND WAVELENGTH ASSIGNMENT PROBLEM (RRWA). If in addition D is symmetric (that is, $(s, t) \in D$ implies $(t, s) \in D$) and the directed paths for requests (s, t) and (t, s) are required to be reverse of each other, then the problem is called the SYMMETRIC RING ROUTING AND WAVELENGTH ASSIGNMENT PROBLEM (SRRWA). A demand is called a permutation demand if, for each vertex v of G, the number of requests with source v and the number of requests with destination v are the same and are equal to 0 or 1. A symmetric permutation demand is called an involution demand. In this paper we prove that both RRWA and SRRWA are $\mathbb{NP}$-complete even when restricted to involution demands. As a consequence RRWA is $\mathbb{NP}$-complete when restricted to permutation demands. For general demands we prove that RRWA and SRRWA can be solved in polynomial time when the number of wavelengths is fixed. Finally, we answer in the negative an open problem posed by Gargano and Vaccaro and construct infinitely many counterexamples using involution demands.

Keywords: Routing; Ring; Wavelength assignment; Permutation; Involution

1. Introduction

The ROUTING AND WAVELENGTH ASSIGNMENT PROBLEM has been receiving extensive attention due to its importance in optical networking and telecommunication. In this paper we study this problem for optical ring networks and answer an open question.

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An optical network can be modelled by an undirected graph $G = (V(G), E(G))$, in which vertices represent processors, routers or memory modules and edges represent optical fibre links. It is usually assumed that each link allows data streams to transmit in both directions. (Physically this is realized by providing two optical fibres.) Thus, we will take G as a *symmetric* directed graph in which there is an *arc* from u to v if and only if there is an arc from v to u, for $u, v \in V(G)$. An ordered pair of distinct vertices of G is called a communication *request* on G, and a set of such requests is called a *demand*. In practice a request (s, t) corresponds to a data stream to be transmitted from s to t, which are called the *source* and *destination* of (s, t) respectively, and a demand corresponds to communications to be realized in one round of routing. Taking each request (s, t) as the arc from s to t, we may identify a demand D on G with a directed graph having the same vertex set $V(G)$ as G. In this directed graph, an isolated vertex is neither a source nor a destination of any request in D, and vice versa. Depending on properties of D, we may have various types of demands. For example, if D is a complete directed graph on $V(G)$ (that is, $D = \{(s, t)\colon s, t \in V(G), s \neq t\}$), then it is an *all-to-all* demand; if D has no isolated vertex and all arcs of D emanate from the same vertex, then it is a *one-to-all* demand. These two types of demands have been well studied in the literature in the context of routing and wavelength assignment; see e.g. [6,14]. A demand on G is said to be a *permutation demand* [6,10,14,20,22] if each connected component of the corresponding directed graph is a directed cycle (possibly with length 2) or an isolated vertex. In other words, a demand is a permutation demand if and only if, for each $v \in V(G)$, the number of requests with source v and the number of requests with destination v are the same and are equal to 0 or 1. A permutation demand D can be identified with the permutation which permutes u to v for each $(u, v) \in D$ and fixes each of the isolated vertices of D. Thus, permutation demands are in one-to-one correspondence with permutations of vertices of G.

Given a demand D on an optical network G, realization of D consists of the following two steps. First, for each $(s, t) \in D$, we need to design a directed path $R(s, t)$ of G from s to t; a set

$$\mathcal{R} = \big\{R(s, t)\colon (s, t) \in D\big\}$$

of such directed paths is called a *routing* for D. For each arc a of G, the *load of a* under $\mathcal{R}$, denoted by $\vec{\pi}(\mathcal{R}, a)$, is the number of directed paths in $\mathcal{R}$ which traverse *a in its direction*. Let

$$\vec{\pi}(\mathcal{R}) = \max\big\{\vec{\pi}(\mathcal{R}, a)\colon a \text{ an arc of } G\big\}$$

be the maximum load of an arc of G under $\mathcal{R}$. Define

$$\vec{\pi}(D) = \min\big\{\vec{\pi}(\mathcal{R})\colon \mathcal{R} \text{ a routing of } D\big\}.$$

In the literature $\vec{\pi}(D)$ is called the *arc-load* [6,9,14,24] of G with respect to D, or the *arc-congestion* [19,23] of (G, D). In the case where D is all-to-all, $\vec{\pi}(D)$ is known as the *arc-forwarding index* [19] of G.

Using the Wavelength Division Multiplexing (WDM) technology, the high bandwidth of optic fibres is partitioned into channels, each of which supports a different wavelength. In this way multiple data streams can be transmitted concurrently along the same fibre provided that they are assigned different wavelengths. In accordance with this, in the second step of realizing a demand D we need to assign a wavelength to each directed path in $\mathcal{R}$ such that two such paths having an arc in common receive distinct wavelengths. Let $\vec{w}(\mathcal{R})$ be the minimum number of wavelengths needed in such an assignment for $\mathcal{R}$. Since wavelengths are limited and costly resources, making effective use of them is an important issue in optical networking. Thus, we define [14]

$$\vec{w}(D) = \min\big\{\vec{w}(\mathcal{R})\colon \mathcal{R} \text{ a routing for } D\big\}$$

and call it the *wavelength number* of D. Given (G, D), the ROUTING AND WAVELENGTH ASSIGNMENT PROBLEM (RWA) seeks a routing $\mathcal{R}$ for D and an assignment of wavelengths to the directed paths in $\mathcal{R}$ such that $\vec{w}(\mathcal{R}) = \vec{w}(D)$, that is, the number of wavelengths used by $\mathcal{R}$ is minimized.

The arc-congestion $\vec{\pi}(D)$ provides a natural lower bound on the wavelength number $\vec{w}(D)$. In fact, since $\vec{w}(\mathcal{R}) \geqslant \vec{\pi}(\mathcal{R}, a)$ for each arc a of G, we have $\vec{w}(\mathcal{R}) \geqslant \vec{\pi}(\mathcal{R})$. Hence, for any demand D on G,

$$\vec{w}(D) \geqslant \vec{\pi}(D).$$

Taking wavelengths as colours, we may interpret wavelength assignments as proper (vertex) colourings of the *conflict graph* $G(\mathcal{R})$, which is defined [14] to have vertices the directed paths in $\mathcal{R}$ such that two such paths are adjacent if