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Gathering six oblivious robots on anonymous symmetric rings ☆,☆☆

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ABSTRACT

A recent model for robot-based computing systems makes use of identical, memoryless, and mobile robots placed on nodes of anonymous graphs. Robots operate in Look-Compute-Move cycles; in one cycle, a robot takes a snapshot of the current robots disposal on the entire ring (Look), takes a decision whether to stay idle or to move to one of its adjacent nodes (Compute), and in the latter case makes a move to this neighbor (Move). Cycles are performed asynchronously for each robot.

We consider the case of six robots placed on the nodes of an anonymous ring in such a way they constitute a symmetric placement with respect to one single axis of symmetry, and we ask whether there exists a strategy that allows the robots to gather at one node. This is the first case left open after a series of papers dealing with the gathering of oblivious robots on anonymous rings. As long as the gathering is feasible, we provide a new distributed approach that guarantees a positive answer to the posed question.

1. Introduction

We study one of the most fundamental problems of self-organization of mobile robots, known in the literature as the *gathering* problem. Robots, initially situated at different locations, have to gather at the same location (not determined in advance) and remain in it. We consider the case of an anonymous ring in which neither nodes nor links have any labels. Initially, some of the nodes of the ring are occupied by robots and there is at most one robot in each node. Robots operate in Look-Compute-Move cycles. In each cycle, a robot takes a snapshot of the current robots disposal on the entire ring (Look), then, based on that, takes a decision to stay idle or to move to one of its adjacent nodes (Compute), and in the latter case makes an instantaneous move to this neighbor (Move). Cycles are performed asynchronously for each robot. This means that the time between Look, Compute, and Move operations is finite but unbounded, and is decided by the adversary for each robot. The only constraint is that moves are instantaneous, and hence any robot performing a Look operation sees all other robots at nodes of the ring and not on edges. However, a robot r may perform a Look operation at some time t, perceiving robots at some nodes, then Compute a target neighbor at some time $t' > t$, and Move to this neighbor at some later time $t'' > t'$, when some robots are in different nodes from those previously perceived by r at time t because in the meantime they performed their Move operations. Hence, robots may move based on significantly outdated perceptions. In fact, robots are memoryless (oblivious), i.e., they do not have any memory of past observations. Thus, the target node (which is either the current position of the robot or one of its neighbors) is decided by the robot during a Compute operation solely on the

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basis of the location of other robots perceived in the previous Look operation. Robots are anonymous and execute the same deterministic algorithm. They cannot leave any marks at visited nodes, nor send any messages to other robots.

We remark that the Look operation provides to a robot the current disposal on the entire ring of any other robot. Moreover, it is assumed that the robots have the ability to perceive whether there is one or more robots located at a given node of the ring. This capability of robots is important and well-studied in the literature under the name of *multiplicity detection* [8–11,14,15,17]. In fact, without this capability, many computational problems (such as the gathering problem considered herein) are impossible to solve for all non-trivial initial disposals of robots. The multiplicity detection capability has been exploited in various forms. In any case, a robot perceives whether a node is empty or not, but in the *global-strong* version, a robot is able to perceive the exact number of robots that occupy each node. In the *global-weak* version, a robot perceives only whether a node is occupied by one robot or if a multiplicity occurs, i.e., a node is occupied by an undefined number of robots greater than one. In the *local-strong* version, a robot can perceive only whether a node is occupied or not, but it is able to perceive the exact number of robots occupying the node where it resides. Finally, in the *local-weak* version, a robot can perceive the multiplicity only on the node where it resides but not the exact number of robots composing it. In this paper, we assume that the robots are empowered with the global weak multiplicity detection capability.

1.1. Related work and our results

Under the Look-Compute-Move model, the gathering problem on rings was initially studied in [15], where certain robots disposals were shown to be gatherable by means of symmetry-breaking techniques, but the question of the general-case solution was posed as an open problem. In particular, it has been proved that the gathering is not feasible with only 2 robots, in *periodic* disposals (invariable under non-trivial rotation) or in those with an axis of symmetry of type edge-edge. A disposal is called *symmetric* if the ring has a geometrical *axis of symmetry*, which reflects single robots into single robots, multiplicities into multiplicities, and empty nodes into empty nodes. A symmetric disposal is aperiodic if and only if it has exactly one axis of symmetry [15]. A symmetric disposal with an axis of symmetry has an *edge-edge symmetry* if the axis goes through (the middles of) two edges; it has a *node-edge symmetry* if the axis goes through one node and one edge; it has a *node-node symmetry* if the axis goes through two nodes; it has a *robot-on-axis symmetry* if there is at least one node on the axis of symmetry occupied by a robot. For an odd number of robots, a gathering algorithm for all the gatherable disposals has been provided. For an even number of robots greater than two, if the initial disposal is aperiodic, the feasibility of the gathering has been solved, except for some types of symmetric disposals. In [14], the attention has been devoted to these left open symmetric cases. The new proposed technique was based on preserving symmetry rather than breaking it, and the problem was solved when the number of robots is greater than 18. This left open the cases of an even number of robots between 4 and 18, as the case of just 2 robots is not gatherable [15]. The case of 4 robots has been solved in [9,16]. Moreover, in [9] all the cases of $2k$ robots with $k \geqslant 2$ have been addressed when the initial axis of symmetry is of type robot-on-axis. Hence, the first case left open concerns 6 robots with an initial axis of symmetry of type node-edge, or node-node. In this paper, we address the problem of 6 robots and provide a distributed algorithm that gathers the robots when starting from any symmetric disposal of type node-edge, or node-node.

Recently in [4] a full characterization of the general problem has been provided and an algorithm that solves all the gatherable disposals has been designed. Such an algorithm makes use of some known techniques as sub-procedures. In particular, for the special cases of 4 and 6 robots, the algorithm relies on the results in [9,16] and this paper, respectively. This confirms the interest in the special cases with a few number of robots. In fact, in such cases, it is easy to incur in not gatherable disposals or generate new axes of symmetry by means of one move. Therefore, neither symmetry breaking nor symmetry preserving techniques can always solve the gathering.

With local weak multiplicity detection capability, an algorithm starting from asymmetric and aperiodic disposals where the number of robots k is strictly smaller than $\lfloor \frac{n}{2} \rfloor$ has been designed in [11]. In [12], the case where k is odd and strictly smaller than $n-3$ has been solved. In [13], the authors provide an algorithm for the case where n is odd, k is even, and $10 \leqslant k \leqslant n-5$. Papers [12] and [13] do not assume that the initial disposal is asymmetric and aperiodic. The asymmetric and aperiodic case has been solved in [6].

Papers [1,7] study the global strong multiplicity detection, while in [2] the gathering in the grid topology has been solved without any multiplicity detection capability but the used techniques cannot be extended to the ring case.

A recent survey on the gathering problem under the Look-Compute-Move model is given in [5].

2. Definitions and notation

We consider an n-node anonymous ring without orientation. Initially, exactly six nodes of the ring are occupied by robots. During a Look operation, a robot perceives the relative locations on the ring of multiplicities and single robots. We remind that a multiplicity occurs when more than one robot occupy the same node.

The current disposal of the robots can be described in terms of the view of a robot r which is performing the Look operation. We denote a disposal seen by r as a tuple $Q(r) = (q_0, q_1, \ldots, q_j)$, $j \leqslant 5$, which represents the sequence of the numbers of empty consecutive nodes interleaved by robots when traversing the ring either in clockwise or in anti-clockwise direction, starting from r. Such a tuple is referred as a *disposal view*. When comparing two disposal views, we decide whether they are equal regardless the traversing orientation. Formally, given two disposal views $Q = (q_0, q_1, \ldots, q_j)$ and