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Behavioural equivalences for coalgebras with unobservable moves

Tomasz Brengos [a,*], Marino Miculan [b], Marco Peressotti [b,*]

[a] Faculty of Mathematics and Information Sciences, Warsaw University of Technology, Poland
[b] Laboratory of Models and Applications of Distributed Systems, Department of Mathematics and Computer Science, University of Udine, Italy

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ABSTRACT

We introduce a general categorical framework for the definition of weak behavioural equivalences, building on and extending recent results in the field. This framework is based on special *order enriched categories*, i.e. categories whose hom-sets are endowed with suitable complete orders. Using this structure we provide an abstract notion of *saturation*, which allows us to define various (weak) behavioural equivalences. We show that the Kleisli categories of many common monads are categories of this kind. On one hand, this allows us to instantiate the abstract definitions to a wide range of existing systems (weighted LTS, Segala systems, calculi with names, etc.), recovering the corresponding notions of weak behavioural equivalences; on the other, we can readily provide new weak behavioural equivalences for more complex behaviours, like those definable on presheaves, topological spaces, measurable spaces, etc.

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1. Introduction

Since Aczel's seminal work [2], the theory of coalgebras has been recognised as a good framework for the study of concurrent and reactive systems [40]: systems are represented as maps of the form $X \to BX$ for some suitable *behavioural functor* B. By changing the underlying category and the functor we can cover a wide range of cases, from traditional labelled transition systems to systems with I/O, quantitative aspects, probabilistic distribution, stochastic rates, and even systems with continuous state. Frameworks of this kind are very useful both from a theoretical and a practical point of view, since they prepare the ground for general results and tools which can be readily instantiated to various cases, and moreover they help us to discover connections and similarities between apparently different notions. In particular, Milner's strong bisimilarity can be characterised by the final coalgebraic semantics and *coalgebraic bisimulation*; this has paved the way for the definition of strong bisimilarity for systems with peculiar computational aspects and many other important results (such as Turi and Plotkin's bialgebraic approach to *abstract GSOS* [50]). More recently, Hasuo et al. [20] have showed that, when the functor B is of the form TF where T is a monad, the *trace equivalence* for systems of the form $X \to TFX$ can be obtained by lifting F to the Kleisli category of T. This has led to many results connecting formal languages, automata theory and coalgebraic semantics [45,44,9,8].

These remarkable achievements have boosted many attempts to cover other equivalences from van Glabbeek's spectrum [51]. However, when we come to behavioural equivalences for systems with unobservable (i.e., internal) moves, the situation

* Corresponding author.
E-mail addresses: t.brengos@mini.pw.edu.pl (T. Brengos), marino.miculan@uniud.it (M. Miculan), marco.peressotti@uniud.it (M. Peressotti).

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is not as clear. The point is that what is "unobservable" depends on the system: in LTSs these are internal steps (the so-called "τ-transitions"), but in systems with quantitative aspects or dealing with resources internal steps may still have observable effects. This has led to many definitions, often quite *ad hoc*. Some follow Milner's "double arrow" construction (i.e., strong bisimulations of the system saturated under τ-transitions), but in general this construction does not work; in particular for quantitative systems we cannot apply directly this schema, and many other solutions have been proposed; see e.g. [7,47,6,31,18,12]. In non-deterministic probabilistic systems, for example, the counterpart of Milner's weak bisimulation is Segala's weak bisimulation [43], which differs from Baier–Hermanns' [5].

This situation points out the need for a general, uniform framework covering many weak behavioural equivalences at once. This is the problem we aim to address in this paper. Analysing previous work in this direction [31,18,10], a common trait we notice is that systems are *saturated* by adding unobservable moves to "fill the gaps" which can be observed by a strong bisimulation. Although different notions of saturations are used for different weak bisimilarities, they are accumulated by *circular* definitions; in turn, these definitions can be described as *equations* in a suitable domain of *approximants* and solved taking advantage of some fixed point theory. Different equations and different domains yield different saturations and hence different notions of weak bisimilarity.

In the wake of these observations, in this paper we propose to host these constructions in *order-enriched categories* whose hom-sets are additionally endowed with *binary joins* for "merging" approximants, and a *complete order* to guarantee convergence of approximant chains. In this setting we can define, and solve, the abstract equations corresponding to many kinds of weak observational equivalence. For example, we will show the abstract schemata corresponding to Milner's and Baier–Hermanns' versions of weak bisimulations; hence, these two different bisimulations are applications of the same general framework. Then, we show that the Kleisli categories of many monads commonly used for defining behavioural functors meet these mild requirements; this allows us to port the definitions above to a wide range of behaviours in different categories (such as presheaf categories, topological spaces and measurable spaces).

Another aspect highlighted by these applications is that the notion of "unobservability" can be considered as a specific computational effect embedded in the monad structure presented by the behavioural functors. This observation fosters the idea that unobservability is orthogonal to saturation, in the sense that we can develop a theory of saturation modularly in the notion of unobservability: which aspects are ignored in the behavioural equivalence can be decided by choosing the suitable unobservation monad.

Synopsis. Section 2 provides some preliminaries about coalgebras, monads and order enriched categories. Section 3 contains the main contribution of this work, i.e. a 2-categorical perspective on weak behavioural equivalence for coalgebras whose behaviours have an unobservable part. In Section 4 we provide a few representative examples of behaviours covered by our results. Some conclusions and directions for further work are in Section 5. Longer proofs are in Appendix A.

We assume the reader to be familiar with the theory of coalgebras and behavioural equivalences; for an introduction, we refer to [3].

2. Preliminaries

2.1. Monads

A monad on a category C is a triple (T, μ, η) where T is an endofunctor over C and $\mu\colon TT \Rightarrow T$ and $\eta\colon \mathcal{I}d \Rightarrow T$ are two natural transformations such that the diagrams below commute:

$$\begin{tikzcd} T^3 \arrow[r, Rightarrow, "\mu T"] \arrow[d, Rightarrow, "T\mu"'] & T^2 \arrow[d, Rightarrow, "\mu"] \\ T^2 \arrow[r, Rightarrow, "\mu"'] & T \end{tikzcd} \qquad \begin{tikzcd} T \arrow[r, Rightarrow, "\eta T"] \arrow[dr, Rightarrow, "\mathcal{I}d"'] & T^2 \arrow[d, Rightarrow, "\mu"] & T \arrow[l, Rightarrow, "T\eta"'] \arrow[dl, Rightarrow, "\mathcal{I}d"] \\ & T & \end{tikzcd}$$

These are the coherence conditions of an associative operation with a unit. In fact, monads over C are monoids in the category of endofunctors over C w.r.t. the monoidal structure defined by endofunctor composition. Hence, μ and η are called *multiplication* and *unit* of T, respectively.

Each monad (T, μ, η) gives rise to a canonical category called *Kleisli category of* T and denoted by $\mathcal{K}l(T)$. This category has the same objects of the category C underlying T; its hom-sets are given as $\mathcal{K}l(T)(X, Y) = \mathsf{C}(X, TY)$ for any two objects X and Y and its composition as $g \bullet f = \mu_Z \circ Tg \circ f$ for any two morphisms f and g with suitable domain and codomain ($\bullet$ and $\circ$ denote composition in $\mathcal{K}l(T)$ and C respectively).

Example 2.1. The powerset functor $\mathcal{P}$ admits a monad structure $(\mathcal{P}, \mu, \eta)$ where $\eta_X(x) = \{x\}$ and $\mu_X(Y) = \bigcup Y$ for any set X.

Example 2.2. The probability distribution functor $\mathcal{D}$ assigns to any set X the set $\mathcal{D}X \triangleq \{\phi\colon X \to [0,1] \mid \sum_{x\in X}\phi(x) = 1\}$ of discrete measures and to any function $f\colon X \to Y$ the function $\mathcal{D}f(\phi)(y) \triangleq \sum_{f(x)=y}\phi(x)$; it admits a monad structure whose unit and multiplication are given on each component X as $\eta_X(x) = \delta_x$ and $\mu_X(\phi)(x) = \sum_\psi \psi(x)\cdot\phi(\psi)$.

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