J. Parallel Distrib. Comput. 74 (2014) 1719–1729

Contents lists available at ScienceDirect

J. Parallel Distrib. Comput.

journal homepage: www.elsevier.com/locate/jpdc

Optimal number of annuli for maximizing the lifetime of sensor networks

Keqin Li

Department of Computer Science, State University of New York, New Paltz, NY 12561, USA

HIGHLIGHTS

- Maximize the lifetime of a wireless sensor network by optimal network design.
- Represent the network lifetime as a function of the number of annuli.
- Find an expression of the optimal number of annuli for an arbitrary sensor density function.

ARTICLE INFO

Article history:
Received 24 August 2012
Received in revised form
15 May 2013
Accepted 24 September 2013
Available online 30 September 2013

Keywords:
Lifetime maximization
Network design
Optimal energy allocation
Wireless sensor network

ABSTRACT

The most effective way to maximize the lifetime of a wireless sensor network (WSN) is to allocate initial energy to sensors such that they exhaust their energy at the same time. The lifetime of a WSN as well as an optimal initial energy allocation are determined by a network design. The main contribution of the paper is to show that the lifetime of a WSN can be maximized by an optimal network design. We represent the network lifetime as a function of the number m of annuli and show that m has significant impact on network lifetime. We prove that if the energy consumed by data transmission is proportional to $d^\alpha + c$, where d is the distance of data transmission and α and c are some constants, then for a circular area of interest with radius R, the optimal number of annuli that maximizes the network lifetime is $m = R((\alpha - 1)/c)^{1/\alpha}$ for an arbitrary sensor density function.

1. Introduction

Wireless sensor networks (WSNs) provide pervasive instrumentation that enables us to observe and interact with the physical and social world and to realize the vision of an embedded Internet. WSNs consisting of mass-produced intelligent sensors have been widely used in environmental and habitat monitoring, climate control, surveillance, intelligent alarms, structural monitoring, ecophysiology, equipment maintenance, medical diagnostics, disaster management, emergence response, asset tracking, healthcare, and manufacturing process flow [9,10].

Due to severe energy constraint in sensors, the lifetime of a WSN has gained substantial research attention [13]. Energy consumption in WSNs contains two components, namely, the energy required for data sensing and the energy used for data transmission. Research in lifetime maximization of WSNs has been focused on the first component only [5,6,11,25], and the second component only [7,12,17,18,22], and both components [1,2,29]. We believe that the lifetime maximization problem of WSNs should be studied by taking both components of energy consumption into consideration [8].

Several methods have been proposed to increase the lifetime of a WSN, including redundant sensors [28], nonuniform sensor distributions [26], and aggregation and forwarding nodes for data transmission [15,27]. All these methods are based on the observation that sensors consume their battery power at different speeds. In particular, sensors close to a base station consume energy much faster than sensors far away from the base station [16,21]. Therefore, the most effective way to maximize the lifetime of a WSN is to allocate initial energy to sensors such that they exhaust their energy at the same time [1,20,23,24].

We find that the lifetime of a WSN as well as an optimal initial energy allocation are determined by a network design. Network lifetime maximization is a two-stage process, namely, optimal network design and optimal energy allocation. In reality, a WSN design includes the locations, sensing ranges, communication ranges, and data generation rates of all sensors, energy consumption for both data sensing and data transmission, as well as a routing algorithm for data transmission to a base station (i.e., a sink). All these factors have impact on sensor and network lifetime as well as optimal energy allocation [20].

The main contribution of the paper is to show that the lifetime of a WSN can be maximized by an optimal network design. By proper modeling and simplification, we represent the network

E-mail address: lik@newpaltz.edu.

0743-7315/$ – see front matter © 2013 Elsevier Inc. All rights reserved.
http://dx.doi.org/10.1016/j.jpdc.2013.09.010

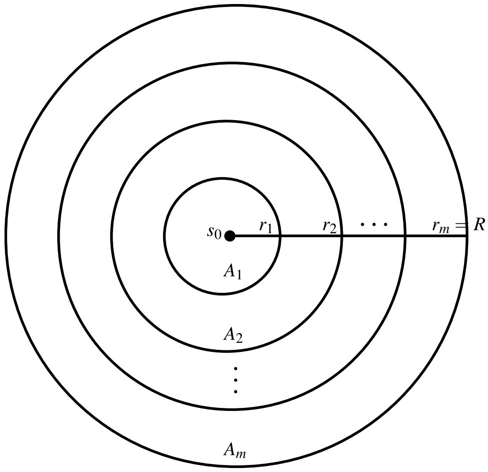

Fig. 1. A circular area of radius R with m annuli.

lifetime obtained by optimal energy allocation as a function of the number m of annuli and show that m has a significant impact on network lifetime. We prove that if the energy consumed by data transmission is proportional to $d^\alpha + c$, where d is the distance of data transmission and α and c are some constants, then for a circular area of interest with radius R, the optimal number of annuli that maximizes the network lifetime is

$$m = R\left(\frac{\alpha - 1}{c}\right)^{1/\alpha},$$

for an arbitrary sensor density function. (Notice that for real applications, m should be rounded to the nearest integer, i.e., either $\lfloor m \rfloor$ or $\lceil m \rceil$; however, we will eliminate such notations for clarity of presentation.)

The organization of the paper is as follows. In Section 2, we present the network model used in our study. In Section 3, we develop analytical forms of network lifetime and optimal energy allocation. In Section 4, we derive the optimal number of annuli for a uniform distribution. In Section 5, we extend our results in Section 3 to arbitrary sensor density functions. In Section 6, we derive the optimal number of annuli for a nonuniform distribution. In Section 7, we demonstrate numerical examples. In Section 8, we prove our general result. We conclude the paper in Section 9.

2. The network model

Let us consider a circular area of interest A which has radius R meters (see Fig. 1). Assume that A is divided into m annuli (also called coronae) $A_1, A_2, \ldots, A_m$ by m circles with radii $r_1, r_2, \ldots, r_m$ centered at a sink, where $0 < r_1 < r_2 < \cdots < r_m = R$ [23]. For convenience, we assume that there is A_0 with width $r_0 = 0$ which contains a sink. All sensors report sensory data to the sink. For a fixed R, the number m of annuli as well as the sequence of values $(r_1, r_2, \ldots, r_{m-1})$ is called a *network design* or a *network configuration*, which has a significant impact on energy consumption and network lifetime.

Annulus A_j has width $r_j - r_{j-1}$, where $1 \le j \le m$. In this paper, we consider the case when all annuli have identical width r, i.e., $r_j - r_{j-1} = r$ for all $1 \le j \le m$. In other words, we have $r_j = jr$, where $r = R/m$.

Assume that there are N sensors $s_1, s_2, \ldots, s_N$ uniformly distributed in A (later, we will consider nonuniform sensor distributions). We use s_0 to represent a sink. All sensors in A_j are designed in such a way that they have the same transmission range $r_j - r_{j-1}$. All sensors also have certain sensing range. It is assumed that N is sufficiently large such that a WSN is connected. Furthermore, it is assumed that the sensing range is sufficiently large such that A is well covered. Let N_j be the number of sensors in A_j. Then, we have

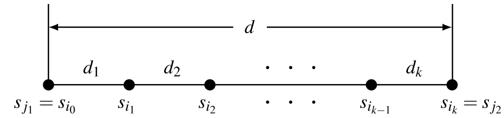

Fig. 2. A data transmission path.

$$N_j = \left(\frac{\pi r_j^2 - \pi r_{j-1}^2}{\pi R^2}\right) N = \left(\frac{r_j^2 - r_{j-1}^2}{R^2}\right) N = \left(\frac{2j-1}{m^2}\right) N,$$

and $N = N_1 + N_2 + \cdots + N_m$.

The amount of energy consumed by a sensor to sense and receive data in one unit of time is p mJ/s.

The amount of energy needed to transmit one bit over distance d meters is $(a_1 d^\alpha + a_2)$ pJ, where a_1 is the energy required to run a transmitter amplifier, a_2 is the energy used to activate a transmitter circuitry, and $2 \le \alpha \le 6$ is a constant [14]. The above expression has significant implication in minimizing energy cost of data transmission in WSNs. Consider a sensor s_{j_1} which sends a bit to another sensor s_{j_2} along a path $(s_{i_0}, s_{i_1}, s_{i_2}, \ldots, s_{i_k})$ with k hops, where $i_0 = j_1$ and $i_k = j_2$ (see Fig. 2). For simplicity, we assume that the $(k+1)$ sensors are on the same line, such that the distance between s_{j_1} and s_{j_2} is d, and the distance between $s_{i_{l-1}}$ and s_{i_l} is d_l, for all $1 \le l \le k$, with $d_1 + d_2 + \cdots + d_k = d$. Then, the energy consumed by the above data transmission is a function of $d_1, d_2, \ldots, d_k$,

$$E(d_1, d_2, \ldots, d_k) = \sum_{l=1}^{k} (a_1 d_l^\alpha + a_2).$$

It has been known that due to the convexity of d^α, the above function is minimized when $d_1 = d_2 = \cdots = d_k = d/k$ [3,23]. Hence, $E(d_1, d_2, \ldots, d_k)$ becomes a function of k,

$$E(k) = k\left(a_1\left(\frac{d}{k}\right)^\alpha + a_2\right) = \frac{a_1 d^\alpha}{k^{\alpha-1}} + a_2 k.$$

It is clear that to minimize $E(k)$, the first term prefers multiple hops of short distance, while the second term prefers a single hop of long distance. The function $E(k)$ is minimized when

$$\frac{dE(k)}{dk} = -\frac{a_1(\alpha-1)d^\alpha}{k^\alpha} + a_2 = 0,$$

that is,

$$k = \left(\frac{a_1}{a_2}(\alpha-1)\right)^{1/\alpha} d,$$

which gives

$$\begin{aligned} E(k) &= \left(\frac{a_1}{a_2}(\alpha-1)\right)^{1/\alpha}\left(\frac{\alpha}{\alpha-1}\right) a_2 d \\ &= a_1^{1/\alpha} a_2^{1-1/\alpha}\left(\frac{\alpha}{(\alpha-1)^{1-1/\alpha}}\right) d. \end{aligned}$$

Such a phenomenon inspires the optimal network configuration problem solved in this paper.

Assume that each datum has size b bytes $= 8b$ bits. Then, the amount of energy needed to transmit one datum over distance d meters is

$$\begin{aligned} q &= 8b(a_1 d^\alpha + a_2)\ \text{pJ} = 8a_1 b\left(d^\alpha + \frac{a_2}{a_1}\right)\ \text{pJ} \\ &= \frac{8a_1 b}{10^6}(d^\alpha + c)\ \text{mJ} = a(d^\alpha + c)\ \text{mJ}, \end{aligned}$$