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Forwarding and optical indices of 4-regular circulant networks

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ABSTRACT

An all-to-all routing in a graph G is a set of oriented paths of G, with exactly one path for each ordered pair of vertices. The load of an edge under an all-to-all routing R is the number of times it is used (in either direction) by paths of R, and the maximum load of an edge is denoted by $\pi(G, R)$. The edge-forwarding index $\pi(G)$ is the minimum of $\pi(G, R)$ over all possible all-to-all routings R, and the arc-forwarding index $\vec{\pi}(G)$ is defined similarly by taking direction into consideration, where an arc is an ordered pair of adjacent vertices. Denote by $w(G, R)$ the minimum number of colours required to colour the paths of R such that any two paths having an edge in common receive distinct colours. The optical index $w(G)$ is defined to be the minimum of $w(G, R)$ over all possible R, and the directed optical index $\vec{w}(G)$ is defined similarly by requiring that any two paths having an arc in common receive distinct colours. In this paper we obtain lower and upper bounds on these four invariants for 4-regular circulant graphs with connection set $\{\pm 1, \pm s\}$, $1 < s < n/2$. We give approximation algorithms with performance ratio a small constant for the corresponding forwarding index and routing and wavelength assignment problems for some families of 4-regular circulant graphs.

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1. Introduction

1.1. Motivation and definitions

Circulant graphs, or multi-loop networks as used in computer science literature, are basic structures for interconnection networks [5]. As such a lot of research on circulant graphs has been done in more than three decades, leading to a number of results on various aspects of circulant graphs [5,11,14,15,18,21,24–27]. Nevertheless, our knowledge on how circulant networks behave with regard to information dissemination is very limited. For example, our understanding to some basic communication-related invariants for circulant graphs such as the arc-forwarding, edge-forwarding and optical indices is quite limited. The purpose of this paper is to study these invariants with a focus on circulant networks of degree 4.

Given an integer $n \geq 3$, denote by $\mathbb{Z}_n$ the group of integers modulo n with operation the usual addition. Given $S \subset \mathbb{Z}_n$ such that $0 \notin S$ and $s \in S$ implies $-s \in S$, the *circulant graph* $C_n(S)$ of order n with respect to S is defined to have vertex set $\mathbb{Z}_n$ such that $i, j \in \mathbb{Z}_n$ are adjacent if and only if $i - j \in S$. (In other words, a circulant graph is a Cayley graph on $\mathbb{Z}_n$.) In the case when $S = \{a, b, n-a, n-b\}$, where $a, b, n-a, n-b$ are pairwise distinct elements of $\mathbb{Z}_n$, $C_n(S)$ is a 4-regular graph (that is, every vertex has degree 4) and we use $C_n(a, b)$ in place of $C_n(S)$. In this paper we deal with circulant graphs $C_n(1, s)$ for some $s \in \mathbb{Z}_n \setminus \{-1, 0, 1, n/2\}$. (Note that when n and a are coprime, $C_n(a, b)$ is isomorphic to $C_n(1, s)$, where $s \equiv a^{-1}b$

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mod n). Without loss of generality, we assume $1 < s < n/2$. Just like any other Cayley graph, $C_n(1, s)$ is vertex-transitive, that is, for any $i, j \in \mathbb{Z}_n$ there exists a permutation of $\mathbb{Z}_n$ that preserves the adjacency relation of $C_n(1, s)$ and maps i to j. (In fact, for fixed i, j this permutation can be chosen as $x \mapsto x + (j - i)$, $x \in \mathbb{Z}_n$ with operation undertaken in $\mathbb{Z}_n$.)

A network can be represented by an undirected graph $G = (V(G), E(G))$, where the node set $V(G)$ represents the set of processors or routers, and the edge set $E(G)$ represents the set of physical links. So we will use the words 'graph' and 'network' interchangeably. We assume the full duplex model, that is, an edge is regarded as two *arcs* with opposite directions over which messages can be transmitted concurrently. A connection request (or a *request* for short) is an ordered pair of distinct nodes (x, y) for which a path $P_{x,y}$ with orientation from x to y in G must be set up to transmit messages from x to y. In this paper we only consider all-to-all communication, or equivalently, the *all-to-all request set* for which one path from every node to every other node must be set up in order to fulfil communications. (In the literature other types of request sets have also been studied.) We call a set of paths $R = \{P_{x,y} : x, y \in V(G), x \neq y\}$ an *all-to-all routing* (or a *routing* for short) in G, where $P_{x,y}$ is not necessarily the same as $P_{y,x}$. The load of an edge e of G with respect to R, denoted by $\pi(G, R, e)$, is the number of paths in R passing through e in either directions. Similarly, the load of an arc a of G with respect to R, denoted by $\vec{\pi}(G, R, a)$, is the number of paths in R passing through a along its direction. Define

$$\pi(G, R) := \max_{e \in E(G)} \pi(G, R, e), \quad \vec{\pi}(G, R) := \max_{a \in A(G)} \vec{\pi}(G, R, a), \tag{1}$$

where $A(G)$ is the set of arcs of G. Define

$$\pi(G) := \min_R \pi(G, R), \quad \vec{\pi}(G) := \min_R \vec{\pi}(G, R) \tag{2}$$

and call them the *edge-forwarding* and *arc-forwarding indices* of G [3,17], respectively, where the minimum is taken over all routings R for G. Obviously, we have

$$\vec{\pi}(G) \geq \pi(G)/2. \tag{3}$$

The *edge-forwarding index problem* is the one of finding $\pi(G)$ for a given graph G, and the *arc-forwarding index problem* is understood similarly.

In practical terms, the edge-forwarding and arc-forwarding indices measure the minimum heaviest load on edges and arcs of a given network, respectively, with respect to all-to-all communication. If the network is all-optical, another important problem is to minimise the number of wavelengths used such that any two paths having an edge (or arc) in common are assigned distinct wavelengths. Regarding wavelengths as colours, these problems can be formulated as the following *path colouring problems*. Given a routing R for G, an assignment of one colour to each path in R is called an *edge-conflict-free colouring* of R if any two paths having an edge in common (regardless of the orientation of the paths) receive distinct colours, and an *arc-conflict-free colouring* of R if any two paths having an arc in common (with the same orientation as the paths) receive distinct colours. (An edge-conflict-free colouring is called *valid* in [12].) Define $w(G, R)$ ($\vec{w}(G, R)$, respectively) to be the minimum number of colours required in an edge-conflict-free (arc-conflict-free, respectively) colouring of R. Define

$$w(G) := \min_R w(G, R), \quad \vec{w}(G) := \min_R \vec{w}(G, R) \tag{4}$$

and call them the *undirected* and *directed optical indices* of G, respectively, where the minimum is taken over all routings R for G. Since the number of colours needed is no less than the number of paths on a most loaded edge (or arc in the directed version), we have (see e.g. [6])

$$w(G) \geq \pi(G), \quad \vec{w}(G) \geq \vec{\pi}(G). \tag{5}$$

In general, equality in (5) is not necessarily true (see e.g. [20,30]). The *routing and wavelength assignment problem* is the problem of computing $w(G)$, and its *oriented version* is the one of finding $\vec{w}(G)$.

1.2. Literature review

The study of the forwarding indices has been intensive in the literature. Heydemann et al. [17] proposed the edge-forwarding index problem and obtained basic results on this invariant, including upper bounds for the Cartesian product of graphs. In [23] it was proved that orbital regular graphs (which are essentially Frobenius graphs [10] except cycles and stars) achieve the smallest possible edge-forwarding index. In [25–27], Thomson and Zhou gave formulas for the edge-forwarding and arc-forwarding indices of two interesting families of Frobenius circulant graphs. The exact value of edge-forwarding index of some other graphs have also been computed, including Knödel graphs [11] and recursive circulant graphs [14]. However, in general it is difficult to find the exact value or a good estimate of the edge-forwarding or arc-forwarding index of a graph, even for some innocent-looking classes of graphs such as circulant graphs. The authors of [29] obtained lower and upper bounds on the edge-forwarding index of a general circulant graph. However, these bounds are difficult to compute in general. Also, a uniform routing of shortest paths may not exist for circulant graphs, just as the case for Cayley graphs in general [16]. The reader is referred to the recent survey [28] for the state-of-the-art on edge-forwarding and arc-forwarding indices of graphs.

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