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Rewriting preserving recognizability of finite tree languages ☆

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ABSTRACT

We show that left-linear generalized semi-monadic TRSs effectively preserve recognizability of finite tree languages (are EPRF-TRSs). We show that reachability, joinability, and local confluence are decidable for EPRF-TRSs.

1. Introduction

The notion of preservation of recognizability through rewriting is a widely studied concept in term rewriting [2–6,8,11–23]. Let Σ be a ranked alphabet, let R be a term rewrite system (TRS) over Σ, and let L be a tree language over Σ. Then $R^*_\Sigma(L)$ denotes the set of descendants of trees in L. A TRS R over Σ preserves Σ-recognizability (is a PΣR-TRS), if for each recognizable tree language L over Σ, $R^*_\Sigma(L)$ is recognizable. A TRS R over Σ preserves Σ-recognizability of finite tree languages (is a PΣRF-TRS), if for each finite tree language L over Σ, $R^*_\Sigma(L)$ is recognizable.

Let R be a TRS over Σ. Then its signature, $sign(R) \subseteq \Sigma$ is the ranked alphabet consisting of all symbols appearing in the rules of R. A TRS R over $sign(R)$ preserves recognizability (is a PR-TRS), if for each ranked alphabet Σ with $sign(R) \subseteq \Sigma$, R, as a TRS over Σ, preserves Σ-recognizability. A TRS R over $sign(R)$ preserves recognizability of finite tree languages (is a PRF-TRS), if for each ranked alphabet Σ with $sign(R) \subseteq \Sigma$, R, as a TRS over Σ, preserves Σ-recognizability of finite tree languages.

A TRS R over Σ effectively preserves Σ-recognizability (is an EPΣR-TRS), if for a given a bottom-up tree automaton (bta) $\mathcal{B}$ over Σ, we can effectively construct a bta $\mathcal{C}$ over Σ such that $L(\mathcal{C}) = R^*_\Sigma(L(\mathcal{B}))$. A TRS R over Σ effectively preserves Σ-recognizability of finite tree languages (is an EPΣRF-TRS), if for a given finite tree language L over Σ, we can effectively construct a bta $\mathcal{C}$ over Σ such that $L(\mathcal{C}) = R^*_\Sigma(L)$. A TRS R over $sign(R)$ effectively preserves recognizability of finite tree languages (is an EPRF-TRS), if for a given ranked alphabet Σ with $sign(R) \subseteq \Sigma$ and a given finite tree language L over Σ, we can effectively construct a bta $\mathcal{C}$ over Σ such that $L(\mathcal{C}) = R^*_\Sigma(L(\mathcal{B}))$.

Gyenizse and Vágvölgyi [13] presented a linear TRS R over $sign(R)$ such that R is an EP$sign(R)$R-TRS and R is not a PR-TRS. A trs R is murg if R is a union of a monadic trs and a right-ground trs. Vágvölgyi [23] showed that it is not decidable for a murg TRS R over Σ whether R is a PΣRF-TRS. Let R be a TRS over $sign(R)$, and let $\Sigma = \{f, \sharp\} \cup sign(R)$, where $f \in \Sigma_2 - sign(R)$ and $\sharp \in \Sigma_0 - sign(R)$. Gyenizse and Vágvölgyi [13] showed that R is an EPΣR-TRS if and only if R is an EPR-TRS. Gyenizse and Vágvölgyi [14] improved this result for left-linear TRSs. They showed the following. Let R be a left-linear TRS over $sign(R)$, and let $\Sigma = \{g, \sharp\} \cup sign(R)$, where $g \in \Sigma_1 - sign(R)$ and $\sharp \in \Sigma_0 - sign(R)$. Then R is an EPΣR-TRS if and only if R is an EPR-TRS.

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In [11] Gilleron showed that for a TRS R over Σ it is not decidable whether R is a PΣR-TRS. We may naturally introduce the above concepts for string rewrite systems as well. Otto [17] has proved that a string rewrite system R over the alphabet $alph(R)$ of R preserves $alph(R)$-recognizability if and only if R preserves recognizability. Otto [17] showed that it is not decidable for a string rewrite system R whether R preserves $alph(R)$-recognizability, and whether R preserves recognizability. Hence it is not decidable for a linear TRS R whether R is a PR-TRS [17].

In spite of the undecidability results of Gilleron [11] and Otto [17], we know several classes of EPR-TRSs. Gyenizse and Vágvölgyi [13] generalized the concept of a semi-monadic TRS [2] introducing the concept of a generalized semi-monadic TRS (GSM-TRS for short). They showed that each linear GSM-TRS R is an EPR-TRS. Takai et al. [19] introduced finite path overlapping TRS's (FPO-TRSs). They [19] showed that each right-linear FPO-TRS R is an EPR-TRS. They [19] also showed that each GSM-TRS R is an FPO-TRS. Thus we get that each right-linear GSM-TRS R is an EPR-TRS. Vágvölgyi [21] introduced the concept of a half-monadic TRS. A trs R over Σ is half-monadic if, for every rule $l \to r$ in R, either $height(r) = 0$ or $r = \sigma(y_1, \dots, y_k)$, where $\sigma \in \Sigma_k$, $k \geq 1$, and for each $i \in \{1, \dots, k\}$, either y_i is a variable (i.e., $y_i \in X$) or y_i is a ground term (i.e., $y_i \in T_\Sigma$). Each right-linear half-monadic TRS is an FPO-TRS. Hence each right-linear half-monadic TRS is an EPR-TRS. Using this result, Vágvölgyi [21] showed that termination and convergence are decidable properties for right-linear half-monadic term rewrite systems. Takai et al. [20] presented an EPR-TRS which is not an FPO-TRS, see Example 1 in [20]. Takai et al. [20] introduced layered transducing term rewriting systems (LT-TRSs). They [20] showed that each I/O separated LT-TRS R is an EPR-TRS.

We show that each terminating TRS is an EPRF-TRS. We adopt the construction of Salomaa [18], Coquidé et al. [2], and Gyenizse and Vágvölgyi [13], when showing that any left-linear GSM-TRS R is an EPRF-TRS. We slightly modify the proofs of the decision results of Gyenizse and Vágvölgyi [13] when we show the following decidability results.

(1) Let R be an EPRF-TRS over Σ, and let $p, q \in T_\Sigma(X)$. Then it is decidable whether $p \to_R^* q$. That is, reachability is decidable.

(2) Let R be an EPRF-TRS over Σ, and let $p, q \in T_\Sigma(X)$. Then it is decidable whether there exists a tree $r \in T_\Sigma(X)$ such that $p \to_R^* r$ and $q \to_R^* r$. That is, joinability is decidable.

(3) Let R be a confluent EPRF-TRS over Σ, and let $p, q \in T_\Sigma(X)$. Then it is decidable whether $p \leftrightarrow_R^* q$. That is, the word problem is decidable.

(4) For an EPRF-TRS R, it is decidable whether R is locally confluent.

(5) Let R be an EPRF-TRS, and let S be a TRS over Σ. Then it is decidable whether $\to_S^* \subseteq \to_R^*$.

(6) Let R and S be EPRF-TRSs. Then it is decidable which one of the following four mutually excluding conditions holds.

(i) $\to_R^* \subset \to_S^*$,
(ii) $\to_S^* \subset \to_R^*$,
(iii) $\to_R^* = \to_S^*$,
(iv) $\to_R^* \bowtie \to_S^*$,

where "$\bowtie$" stands for the incomparability relationship.

(7) Let R be an EPRF-TRS. Then it is decidable whether R is left-to-right minimal. (A TRS R is left-to-right minimal if for each rule $l \to r$ in R, $\to_{R-\{l\to r\}}^* \subset \to_R^*$.)

(8) Let R and S be TRSs such that $R \cup R^{-1}$ and $S \cup S^{-1}$ are EPRF-TRSs. Then it is decidable which one of the following four mutually excluding conditions holds.

(i) $\leftrightarrow_R^* \subset \leftrightarrow_S^*$,
(ii) $\leftrightarrow_S^* \subset \leftrightarrow_R^*$,
(iii) $\leftrightarrow_R^* = \leftrightarrow_S^*$,
(iv) $\leftrightarrow_R^* \bowtie \leftrightarrow_S^*$.

Fülöp's [6] undecidability results on deterministic top-down tree transducers simply imply the following. Each of the following questions is undecidable for any convergent left-linear EPRF-TRSs R and S over a ranked alphabet Ω, for any recognizable tree language $L \subseteq T_\Omega$ given by a tree automaton over Ω recognizing L. Here $\Gamma \subseteq \Omega$ is the smallest ranked alphabet for which $NF_R(L) \subseteq T_\Gamma$. Furthermore, the set of R-normal forms of the trees in L is denoted by $NF_R(L)$.

(i) Is $NF_R(L) \cap NF_S(L)$ empty?
(ii) Is $NF_R(L) \cap NF_S(L)$ infinite?
(iii) Is $NF_R(L) \cap NF_S(L)$ recognizable?
(iv) Is $T_\Gamma - NF_R(L)$ empty?
(v) Is $T_\Gamma - NF_R(L)$ infinite?
(vi) Is $T_\Gamma - NF_R(L)$ recognizable?
(vii) Is $NF_R(L)$ recognizable?
(viii) Is $NF_R(L) = NF_S(L)$?
(ix) Is $NF_R(L) \subseteq NF_S(L)$?

Fülöp and Gyenizse [7] showed that it is undecidable for a tree function induced by a deterministic homomorphism whether it is injective. Hence for any convergent left-linear EPRF-TRS R over a ranked alphabet Σ, and any recognizable tree language $L \subseteq T_\Sigma$, it is undecidable whether the tree function $\to_R^* \cap (L \times NF_R(L))$ is injective.