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Inference engine based on closure and join operators over Truth Table Binary Relations

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ABSTRACT

We propose a conceptual reasoning method for an inference engine. Starting from a knowledge base made of decision rules, we first map each rule to its corresponding Truth Table Binary Relation (TTBR), considered as a formal context. Objects in the domain of TTBR correspond to all possible rule interpretations (in terms of their truth value assignments), and elements in the range of TTBR correspond to the attributes. By using the 'natural join' operator in the 'ContextCombine' Algorithm, we combine all truth tables into a global relation which has the advantage of containing the complete knowledge of all deducible rules. By conceptual reasoning using closure operators, from the initial rules we obtain all possible conclusions with respect to the global relation. We may then check if expected goals are among these possible conclusions. We also provide an approximate solution for the exponential growth of the global relation, by proposing modular and cooperative conceptual reasoning. We finally present experimental results for two case studies and discuss the effectiveness of our approach.

1. Introduction

Automated reasoning has applications in domains such as automated theorem proving [1], software verification [2] and model finding [3]. Automated reasoning also plays an important role in expert systems [4,5], where techniques such as forward chaining, backward chaining, mixed or structural approaches are employed [6]. In this paper, for each decision rule involving some set of terms or attributes P, we create its truth table as a binary relation R over all involved attributes. We then combine all tables in a single one using our proposed 'ContextCombine' Algorithm. The latter explores all possibilities between the different assignments to generate a global solution in a new truth table. By using a Galois connection on the entire table thus obtained, we are able to infer all possible conclusions related with some input facts. Furthermore, we are able to regenerate all implications from this new context [7]. This procedure may produce tables whose size grows beyond reasonable limits. To cope with this limitation of the technique we propose a modular and cooperative reasoning approach that delays combination operations, by first reducing the different initial contexts associated with the different rules with respect to the facts initially submitted to the inference engine.

The article is organized as follows. In Section 2 we present the foundations of formal concept analysis. In Section 3 we propose *conceptual reasoning*, a reasoning method based on Galois connection. In Section 4 we propose a modular

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and cooperative reasoning approach which allows us to partially overcome the limitations of conceptual reasoning. Initial experimentation using this approach shows its efficiency in the two case studies (cf. Section 5.1 and Section 5.2) related to SAT/UNSAT problems and Medical Data, respectively. Finally, in Section 6, we draw some conclusions and present some proposals for further work.

2. Formal Concept Analysis and Relational Algebra

We first recall some basic notions from Formal Concept Analysis (FCA) [8,9] and Relational Algebra [10].

Definition 1. Let $\mathcal{O}$ and $\mathcal{P}$ be sets, called the set of objects and attributes, respectively. Let $\mathcal{R}$ be a relation on $\mathcal{O} \times \mathcal{P}$. For $o \in \mathcal{O}$ and $p \in \mathcal{P}$, $\mathcal{R}(o, p)$ holds if the object o has attribute p, denoted also by $(o, p) \in \mathcal{R}$. The triple $\mathcal{K} = (\mathcal{O}; \mathcal{P}; \mathcal{R})$ is called a *formal context*.

We may notice that the definition of a formal context is very similar to a relation where $\mathcal{O}$ (respectively, $\mathcal{P}$) represents the domain (respectively, the range) of the relation.

2.1. Galois connections and their properties

Definition 2. Let $(A, \leqslant_A)$ and $(B, \leqslant_B)$ be two partially ordered sets. Let $f : A \to B$ and $g : B \to A$ such that $\forall a \in A, b \in B$, $f(a) \leqslant_B b \iff g(b) \leqslant_A a$. Then, the pair (f, g) is called a *Galois connection*.

Proposition 3. *Let $\mathcal{O}$, $\mathcal{P}$ be two arbitrary sets. Let $\mathcal{R}$ be a relation on $\mathcal{O} \times \mathcal{P}$. Let $A \subseteq \mathcal{O}$ and $B \subseteq \mathcal{P}$ also be arbitrary. The pair of functions (f, g) with $f : 2^{\mathcal{O}} \to 2^{\mathcal{P}}$ and $g : 2^{\mathcal{P}} \to 2^{\mathcal{O}}$ defined by*

- $f(A) = \{p \in \mathcal{P} \mid \forall o \in A, (o, p) \in \mathcal{R}\}$,
- $g(B) = \{o \in \mathcal{O} \mid \forall p \in B, (o, p) \in \mathcal{R}\}$,

forms a Galois connection.

Let $A_1, A_2 \subseteq \mathcal{O}$ and $B_1, B_2 \subseteq \mathcal{P}$. It is well known [9] that a pair (f, g) forms a Galois connection if and only if the following properties are satisfied:

$$\begin{aligned} &A_1 \subseteq A_2 \quad \Rightarrow \quad f(A_2) \subseteq f(A_1), && B_1 \subseteq B_2 \quad \Rightarrow \quad g(B_2) \subseteq g(B_1), \\ &A \subseteq (g \circ f)(A), && B \subseteq (f \circ g)(B). \end{aligned}$$

Definition 4. We call $(g \circ f)(A)$ the *closure* of A, and $(f \circ g)(B)$ the *closure* of B. The pair (A, B), where A is included in $\mathcal{O}$, B is included in $\mathcal{P}$, $f(A) = B$, and $g(B) = A$ is called a *formal concept* of context $\mathcal{K}$ with extent A and intent B. We also have $(g \circ f)(A) = A$ and $(f \circ g)(B) = B$.

2.2. Rule representation and reasoning

We are mainly concerned with knowledge base representation and automated reasoning. In this section we show how a truth table associated with a decision rule is represented as a formal context, and how different rules may be combined if there is no contradiction between them.

2.2.1. Rule representation

A decision rule of the form '*if A then B*' or equivalently '$A \to B$ is true', between attributes A and B, reflects the knowledge of a given domain in which the satisfaction of premise A implies the conclusion B. From a logical point of view, the expression '$A \to B$ is true' can be represented as a Truth Table (TT) where each row (or solution) is a truth value assignment ($1 = $ *true*, $0 = $ *false*) for the attributes A and B. For instance, the assignment $(A = 0, B = 1)$ is a solution for '$A \to B$ is true'. At the same time, the later expression could be also represented as a Formal Context (FC) $\mathcal{K} = (\mathcal{O}; \mathcal{P}; \mathcal{R})$, where the set $\mathcal{O}$ is the set of possible solutions (truth-value assignments for A and B such that $A \to B$ is true), the set $\mathcal{P}$ is the set of attributes (or properties, in our case $\mathcal{P} = \{A, B\}$), and $\mathcal{R}(o, p)$ holds if the solution $o \in \mathcal{O}$ has the assignment *true* for the attribute $p \in \mathcal{P}$. In the remainder, the Truth Table Binary Relation $\mathcal{R}$ will be denoted by TTBR and the expression '$A \to B$ is true' will be abbreviated as '$A \to B$'.

Example 5. Let us consider the rule $A \to B$. In Table 1 we find a representation of a formal context $\mathcal{K} = (\mathcal{O}; \mathcal{P}; \mathcal{R})$, where $\mathcal{O} = \{s_1, s_2, s_3\}$, $\mathcal{P} = \{A, B\}$ and $\mathcal{R} = \{(s_1, A), (s_1, B), (s_2, B)\}$.