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Adaptive time-varying detrended fluctuation analysis

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HIGHLIGHTS

- DFA is often used to estimate the scaling exponent of neurophysiological signals.
- We extend DFA to characterise potentially changing scaling exponents.
- We validate the method using surrogate data with time-varying scaling exponents.
- We systematically examine the dependence of the method on its free parameters.
- We demonstrate the applicability of the method to neurophysiological signals.

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ABSTRACT

Detrended fluctuation analysis (DFA) is a technique commonly used to assess and quantify the presence of long-range temporal correlations (LRTCs) in neurophysiological time series. Convergence of the method is asymptotic only and therefore its application assumes a constant scaling exponent. However, most neurophysiological data are likely to involve either spontaneous or experimentally induced scaling exponent changes. We present a novel extension of the DFA method that permits the characterisation of time-varying scaling exponents. The effectiveness of the methodology in recovering known changes in scaling exponents is demonstrated through its application to synthetic data. The dependence of the method on its free parameters is systematically explored. Finally, application of the methodology to neurophysiological data demonstrates that it provides experimenters with a way to identify previously un-recognised changes in the scaling exponent in the data. We suggest that this methodology will make it possible to go beyond a simple demonstration of the presence of scaling to an appreciation of how it may vary in response to either intrinsic changes or experimental perturbations.

1. Introduction

Neurophysiological processes are interaction dominated such that the operation of one component of the system closely depends on the state of another component. In contrast to systems dominated by additive and subtractive interactions, which produce distributions characterised by Gaussian statistics, neuronal activity is characterised by multiplicative interactions that can produce heavy-tailed distributions including power law distributions (Kello et al., 2010).

It has now been established that many neurophysiological signals show power law distributions of their autocovariance function, i.e., they exhibit long-range temporal correlations (LRTCs). These LRTCs have been observed in fluctuations of amplitudes (e.g., Linkenkaer-Hansen et al., 2001, 2004; Nikulin and Brismar, 2005; Berthouze et al., 2010) and inter-event intervals (Hartley et al., 2012). The detection and characterisation of LRTCs in neurophysiological data has received great attention in part due to the fact that LRTCs are a (necessary, but not sufficient) signature of critical systems. The idea that the brain may be operating in a critical regime is very attractive (Chialvo, 2010) because critical systems have been shown to maximise their dynamic range of processing (Kinouchi and Copelli, 2006; Shew et al., 2009; Buckley and Nowotny, 2011), and implement balanced activity (Benayoun et al., 2010; Magnasco et al., 2009; Meisel and Gross, 2009). In their seminal work, Linkenkaer-Hansen et al. (2001) interpreted the presence of LRTCs in the fluctuations of EEG and MEG amplitude oscillations within the framework of criticality in which once LRTCs are established, the scaling exponent would be expected to be constant throughout a normal resting state neurophysiological record. From the perspective of criticality (in its physics sense of the term),

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such an assumption may be justified. However, there is an alternative view which is that heavy-tailed distributions (including power laws) could also be observed as the result of the superposition of processes with distinct time scales (Wagenmakers et al., 2004), or as a result of measurements (Touboul and Destexhe, 2010). In this view, the validity of the assumption that the scaling exponent is constant throughout a neurophysiological recording should be firmly established because the overall organisation of these different time scales may no longer result from a global order parameter. We suggest that any exponent estimation method should be agnostic to the origin of the LRTCs and instead focus on providing a robust estimation of exponent magnitude over small enough time scales within which exponent magnitude fluctuations if present can be observed.

Furthermore, LRTCs in neurophysiological time series have been characterised using estimates of the Hurst exponent which quantifies the slope of the auto-covariance function of the signal, with exponents in the interval (0.5,1] denoting the presence of LRTCs. These estimates can be obtained using several methodologies, see Taqqu and Teverovsky (1995) and Serinaldi (2010) for comprehensive comparative reviews of methods operating in both time and frequency domains, including detrended fluctuation analysis (DFA, Peng et al., 1994). These methodologies estimate the statistical properties of the data under the implicit assumption of constancy of the scaling properties of the signal. Therefore, they are by definition insensitive to any within time series change in the exponent magnitude that characterises LRTCs.

In the case of DFA, which has been extensively used in the neurophysiology literature, if the changes are small enough, the scaling property of the detrended fluctuations can be maintained (based on the R^2 value of the linear regression being greater than a given threshold, typically 0.95) and therefore the method, as commonly implemented in published reports, will return valid exponents without any indication that the assumption of scaling exponent constancy has been violated. Only close inspection or a more robust test of the distribution of the fluctuations in the log–log scale could provide an indication of superposition of processes (Chen et al., 2002; Hu et al., 2001).

To date, there have been a few attempts to track changes in the scaling parameter and these attempts have relied on a rolling implementation of standard DFA methodologies over moving windows (e.g., Alvarez-Ramirez et al., 2008; Peña et al., 2009; Yue et al., 2010). This approach does not involve optimal filtering and has not been validated against time series in which the magnitude of the scaling exponent is systematically manipulated within the record. Further, this approach when applied to non-physiological time-series has been shown to lead to erratic behaviour in the estimates of the scaling exponent (Alvarez-Ramirez et al., 2008; Peña et al., 2009).

Here, we present a novel extension of the detrended fluctuation analysis method (adaptive time-varying detrended fluctuation analysis – ATvDFA) which permits the robust characterisation of time-varying scaling parameters. We systematically compare the ATvDFA method with a moving windows DFA using synthetic data and demonstrate its applicability within 3 different types of neurophysiological time series.

2. Material and methods

2.1. Method formulation

The core component of the method is detrended fluctuation analysis, and it is briefly summarised here. We assume a bounded time series $x(i)$, where $i = \{1, \ldots, N\}$, and N is the length of the signal. First, we construct the integrated signal $y(i)$ as the cumulated sum:

$$y(i) = \sum_{j=1}^{i} (x(i) - \bar{x}) \tag{1}$$

We then construct a set of box sizes $s(k)$ with $k = \{1, \ldots, n\}$ that are equidistant in logarithmic space where n is suitably large to provide enough resolution in the interval $[s(1), s(n)]$, with $s(1)$ and $s(n)$ the inner and outer cut-offs, chosen to maximise the range of temporal correlations whilst providing a sufficiently high number of non-overlapping segments for all box size (Peng et al., 1994). For each box size $s(k)$, the integrated signal is then split into $\lfloor N/s(k) \rfloor$ non-overlapping segments, where $\lfloor x \rfloor$ denotes the largest integer not greater than x. The signal is then locally detrended by subtracting a polynomial fit $\hat{y}(i)$. Finally, for each box size $s(k)$, the root mean square fluctuation for the detrended integrated signal is computed:

$$F(s(k)) = \sqrt{\sum_{i=1}^{N} (y(i) - \hat{y}(i))^2} \tag{2}$$

For signals with long-range temporal correlations, there is a power-law relationship between the root mean square fluctuation $F(s(k))$ and $s(k)$:

$$F(s(k)) \approx s(k)^{\alpha} \tag{3}$$

where α is the scaling exponent and is readily obtained by linear regression of the log detrended fluctuations over the log box sizes. The exponent is accepted if the R^2 value is sufficiently high (typically >0.95) and there is no cross-over in the linear scaling of the log detrended fluctuations in relation to the log box sizes (Chen et al., 2002). Convergence of the method is asymptotic only in the limit of N, the number of samples (Bardet, 2008; Taqqu and Teverovsky, 1995), and therefore the recommended practice is that it should be applied to lengthy time series under the implicit assumption of a constant scaling exponent. However, it has been recently suggested that robust estimates can be obtained even with extremely short time series, especially if the data have genuine long-range correlations (Crevecoeur et al., 2010).

The simplest solution to the problem of detecting changes would be to compute DFA within a moving window (we will refer to this method as mDFA henceforth). Such an approach has been used with non-physiological data previously but leads to considerable statistical variation in the estimates of the scaling exponent (Peña et al., 2009). For short time-series, the application of linear regression of the log fluctuations over the log box size does not lead to a robust estimate of the exponent because of the violation of homoscedasticity, i.e., the fact that the variance in the fluctuations at each box is not identical for all box sizes. Here, we address this problem through the application of a Kalman filter, a data-adaptive filtering procedure, in order to track exponent estimates obtained from overlapping data segments.

A Kalman filter operates over a state-space model, with state and measurement equations given by

$$x_{k+1} = \phi_k x_k + w_k \tag{4}$$

$$z_{k+1} = H_k x_k + v_k \tag{5}$$

where x_k and z_k are the state and measurement vectors, ϕ_k is the state transition matrix, H_k is the state-to-measurement matrix, and w_k and v_k are the process and measurement noise sources respectively. Here, we define the state as the parameters of the linear regression of the log detrended fluctuations $\log F(s(k))_{k=\{1,\ldots,n\}}$ over the log box sizes $\log s(k)_{k=\{1,\ldots,n\}}$. The state vector x_k is therefore defined as the 2×1 column vector $x_k = [u_1, u_2]$ where u_1 is the