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A formal and data-based comparison of measures of motor-equivalent covariation

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ABSTRACT

Different analysis methods have been developed for assessing motor-equivalent organization of movement variability. In the uncontrolled manifold (UCM) method, the structure of variability is analyzed by comparing goal-equivalent and non-goal-equivalent variability components at the level of elemental variables (e.g., joint angles). In contrast, in the covariation by randomization (CR) approach, motor-equivalent organization is assessed by comparing variability at the task level between empirical and decorrelated surrogate data. UCM effects can be due to both covariation among elemental variables and selective channeling of variability to elemental variables with low task sensitivity ("individual variation"), suggesting a link between the UCM and CR method. However, the precise relationship between the notion of covariation in the two approaches has not been analyzed in detail yet.

Analysis of empirical and simulated data from a study on manual pointing shows that in general the two approaches are not equivalent, but the respective covariation measures are highly correlated ($\rho > 0.7$) for two proposed definitions of covariation in the UCM context. For one-dimensional task spaces, a formal comparison is possible and in fact the two notions of covariation are equivalent. In situations in which individual variation does not contribute to UCM effects, for which necessary and sufficient conditions are derived, this entails the equivalence of the UCM and CR analysis. Implications for the interpretation of UCM effects are discussed.

1. Introduction

It has been proposed that successful performance of motor tasks involving abundant degrees of freedom (DOF) depends on motor-equivalent organization of movement variability (Schöner, 1995; Todorov and Jordan, 2002; Latash et al., 2007). The notion of motor equivalence refers to the fact that a particular state of a task variable (e.g., position of an end-effector) can be achieved by a variety of configurations of elemental variables (e.g., joint angles). The present paper compares two quantitative methods that allow analyzing the use of motor-equivalence in functional tasks, the uncontrolled manifold (UCM) and the covariation by randomization (CR) method. Both approaches require the definition of a forward model, i.e. a mapping from elemental to task variables, which relates fluctuations in elemental variables to fluctuations in the task variable.

In the uncontrolled manifold analysis (Scholz and Schöner, 1999), variability in elemental variables is decomposed into goal-equivalent and non-goal-equivalent variability components (GEV and NGEV) with respect to hypothesized task variables. The decomposition of variability is achieved by linearizing the forward model. GEV and NGEV are normalized with respect to the number of DOF. A task variable is controlled (stabilized) in the UCM sense, when GEV exceeds NGEV. UCM effects can be quantified by using a derived measure, e.g., the ratio between GEV and NGEV, or the normalized difference (Latash et al., 2007). It has been noted that comparisons between different conditions or task variables should not be based on the variability structure of elemental variables alone, but should also assess potential differences in the mapping from elemental to task variables (Cusumano and Cesari, 2006; de Freitas et al., 2010).

In contrast to the UCM analysis, the covariation by randomization (CR) approach (Müller and Sternad, 2003) assesses motor-equivalent organization by comparing variability in task space, between empirical and "decorrelated" surrogate data. Decorrelation can be achieved by randomly permuting the elemental variables across samples, thereby removing all possible covariation among them, and then applying the forward model to determine the corresponding variability in task space. Alternatively, for (approximately) linear forward models, the effect of decorrelation can also be analyzed by setting certain entries of the covariance matrix of the elemental variables to zero (Yen and Chang, 2010; Verrel et al., 2010b). Covariation is present when task variability is higher for decorrelated than for empirical variability. A covariation index can be defined as the ratio between decorrelated and empirical variability.

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Thus, the UCM and CR approach address a similar question with different means (Schöner and Scholz, 2007). Recently, it has been noted that UCM effects can be due to both covariation (in the CR sense) and "individual variation", i.e. unequal variance distribution in elemental variables (Schöner and Scholz, 2007; Yen and Chang, 2010). Yen and Chang (2010) proposed a way of separating these two contributions of UCM effects. However, the relationship between the UCM and CR analysis remains insufficiently understood. After briefly introducing the UCM and CR analysis, I discuss different measures of covariation in the UCM context. For one-dimensional task spaces (representing a substantial proportion of published studies using either the CR or UCM method), a formal comparison of covariation criteria in the UCM and CR context is possible. For higher-dimensional task spaces, this equivalence between the two methods does not hold in general. The formal analysis is illustrated by empirical and simulated data for one-dimensional and three-dimensional task spaces.

2. Formal analysis

2.1. Forward model

Both the UCM and the CR analysis are based on a forward model $f: E \to T$, mapping elemental variables (represented in E) to task variables (represented in T). Each point in E represents one configuration of elemental variables, such as a posture defined in terms of joint angles, or a muscle activation pattern represented as a superposition of muscle "modes" (Latash et al., 2007). In general, E and T are finite-dimensional manifolds. However, variability in both elemental and task variables is typically small, allowing to choose local linear representations of E and T (as vector spaces) and to approximate f by its Jacobian $J = Df$. If E is n-dimensional and T is d-dimensional, J is a $d \times n$ matrix of rank d, (provided f is non-degenerate). A given small deviation in elemental variables $e \in E$ is mapped to a deviation in task space $Je \in T$.

Let $e_i \in E$, $i = 1, \ldots, N$ denote the fluctuations (deviations from their mean) in the observed data sample of size N. By a change of coordinate system, one can assume that mean$(e_i) = 0$ and $f(0) = 0$. Using the linear approximation and after coordinate change, one obtains

$$f(e) = Je + O(\|e\|^2),$$

where $O(||e||^2)$ indicates that the non-linearity is bounded above by a constant multiple of $||e||^2$ for $||e||$ close to zero. The UCM analysis is based on this linear approximation. The CR analysis is in principle possible for arbitrary forward models. However, for small fluctuations in elemental variables and forward models that are locally well linearly approximated, the computational requirements of the CR analysis can be greatly reduced by using the approximation.

2.2. Uncontrolled manifold (UCM)

In the UCM analysis, the total variability (TOTV) in E is decomposed into GEV and NGEV by projecting the deviations e_i to the null-space of J, and its orthogonal complement, respectively. These projections are null$(J)^{\mathbf{t}}$ and orth$(J^{\mathbf{t}})^{\mathbf{t}}$, where, for a matrix M, orth(M) and null(M) denote matrices whose column vectors form orthonormal bases for the range and null space of M.

Yen and Chang (2010) provided an elegant computation for the variability components in the UCM analysis, based on the covariance matrix C of the e_i:

$$\mathrm{TOTV} = \frac{\mathrm{trace}(C)}{n}, \tag{1}$$

$$\mathrm{NGEV} = \frac{\mathrm{trace}(\mathrm{orth}(J^{\mathbf{t}})^{\mathbf{t}} \cdot C \cdot \mathrm{orth}(J^{\mathbf{t}}))}{d}, \tag{2}$$

$$\mathrm{GEV} = \frac{\mathrm{trace}(\mathrm{null}(J)^{\mathbf{t}} \cdot C \cdot \mathrm{null}(J))}{n-d}. \tag{3}$$

For consistency with the original UCM analysis, the biased covariance matrix is used (normalized by N rather than $N-1$). It is important to emphasize that each of these components is normalized by the number of DOF. In particular TOTV is not the (unnormalized) total variance but the total variance per DOF, and it is not the arithmetic sum of GEV and NGEV. Instead, the following equation follows from the normalization:

$$n\mathrm{TOTV} = d\mathrm{NGEV} + (n-d)\mathrm{GEV}. \tag{4}$$

By definition (Latash et al., 2007; Scholz and Schöner, 1999), a UCM-effect is present when any of the following equivalent conditions is satisfied

$$\begin{aligned} \mathrm{GEV} > \mathrm{NGEV} \quad &\Leftrightarrow \quad \mathrm{GEV} > \mathrm{TOTV} \\ &\Leftrightarrow \quad \mathrm{NGEV} < \mathrm{TOTV}. \end{aligned} \tag{5}$$

The equivalence of these conditions follows from Eq. (4).

The following two measures have been proposed to quantify UCM effects (e.g., Latash et al., 2007):

$$S = \frac{\mathrm{GEV}}{\mathrm{NGEV}}, \tag{6}$$

$$T = \frac{\mathrm{GEV} - \mathrm{NGEV}}{\mathrm{TOTV}} = \frac{n\mathrm{TOTV} - n\mathrm{NGEV}}{(n-d)\mathrm{TOTV}} \tag{7}$$

(the last equality follows by solving Eq. (4) for GEV and inserting it in the definition of T).

Following Eq. (5), a UCM effects is present when the following conditions hold for S and T:

$$S > 1 \quad \Leftrightarrow \quad T > 0. \tag{8}$$

2.3. Covariation by randomization (CR)

In the covariation by randomization (CR) analysis (Müller and Sternad, 2003), elemental variables are randomly permuted across samples to produce (approximately) covariation-free data (Müller and Sternad, 2003). If the given sample $e_i \in E$ has components

$$e_i = (e_{ij}) \quad \text{with } i = 1, \ldots, N,\ j = 1, \ldots, n,$$

a randomized sample is defined as

$$e_i^{\pi} = (e_{\pi_j(i),j}) \quad \text{with } i = 1, \ldots, N,\ j = 1, \ldots, n,$$

where π is an n-tuple of permutations on N elements,

$$\pi = (\pi_j) \quad \text{with } \pi_j \in \Sigma_N, \quad j = 1, \ldots, n.$$

Task variability for empirical and decorrelated data is defined using the forward model

$$\mathrm{TV} = \mathrm{var}(f(e_i)), \tag{9}$$

$$\mathrm{TV}_{\pi} = \mathrm{var}(f(e_i^{\pi})), \tag{10}$$

where var denotes the ordinary variance for one-dimensional task spaces, and the trace of the covariance matrix for higher-dimensional task spaces. By averaging TV_{π} over many repetitions of the randomization procedure (with different choices of π), task variability for covariation-free data can be estimated. Note that replicability of this procedure is compromised by the (necessary) choice of random permutations.

Alternatively, when f is locally linear, task variability can be computed from the covariance matrix $C = \mathrm{cov}(e_i)$:

$$\mathrm{TV} = \mathrm{var}(Je_i) = \mathrm{trace}(\mathrm{cov}(Je_i)) = \mathrm{trace}(JCJ^{\mathbf{t}}). \tag{11}$$