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Multi-periodic neural coding for adaptive information transfer ☆

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ABSTRACT

Information processing in the presence of noise has been a key challenge in multiple disciplines including computer science, communications, and neuroscience. Among such noise-reduction mechanisms, the shift-map code represents an analog variable by its residues with respect to distinct moduli (that are chosen as geometric scalings of an integer). Motivated by the multi-periodic neural code in the entorhinal cortex, i.e., the coding mechanism of grid cells, this work extends the shift-map code by generalizing the choices of moduli. In particular, it is shown that using similarly sized moduli (for instance, evenly and closely spaced integers, which tend to have large co-prime factors) results in a code whose codewords are separated in an interleaving way such that when the decoder has side information regarding the source, then error control is significantly improved (compared to the original shift map code). This novel structure allows the system to dynamically adapt to the side information at the decoder, even if the encoder is not privy to the side information. A geometrical interpretation of the proposed coding scheme and a method to find such codes are detailed. As an extension, it is shown that this novel code also adapts to scenarios when only a fraction of codeword symbols is available at the decoder.

1. Introduction

The brain represents, stores, and computes with analog variables (e.g., determining and storing the orientation of an edge in the visual world, estimating and representing the speed of motion of a target, comparing the hue of an item with that of another) in the presence of noise in the basic processes of synaptic communication and response generation [1–3]. In these neural computations, representing one variable with a large number of neurons reduces the effects of the noise. In many brain areas, the mean firing rates of neurons vary in characteristic ways with the value of the represented variable, and these functions are called tuning curves. Many neurons in the sensory and the motor cortices have unimodal (or single-bump)

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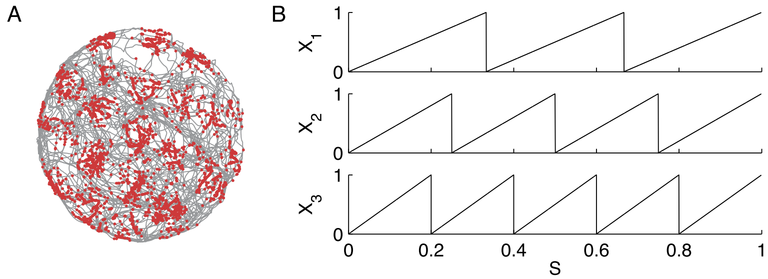

Fig. 1. The response of a grid cell in the medial entorhinal cortex (mEC) is a periodic function of the animal location (A). Groups of grid cells (X_1, X_2, and X_3) encode the spatial location (S) with different periodicities (B).

tuning curves peaked at a certain value of the represented variable. The peak values of different neurons tile the range of the variable. This redundant representation, known as a population code, enables noise tolerance [4–7].

In this work, we investigate the features of a code that is believed to underlie navigational computations in the brain. Our focus is on the inherent neural diversity of the tuning curve functions, and the functional advantage of this diversity from a coding-theoretic viewpoint. The hippocampus, which has long been implicated in spatial learning and memory functions and with the writing of new episodic memories, exhibits largely unimodal tuning curves for the 2-dimensional coordinate that represents the animal's location in space [8]. Grid cells [9], the focus of the present work, reside in medial entorhinal cortex (mEC), a high-order association area that is the main cortical input to hippocampus. Individual grid cells represent animal location with an interesting multi-peaked firing pattern, in which the peaks are arranged on every vertex of a virtual triangular lattice that tiles the explored space, Fig. 1 [9]. Grid cells are organized in a number of distinct functional modules: in each module, grid cells have a common spatial firing period and orientation, but a diversity of spatial phases (that is, tuning curves of different cells in a module are rigid shifts of a canonical lattice pattern). Different modules exhibit different spatial periods. Neurophysiological experiments show 4–5 (and no more than 10) distinct modules, with an approximately geometric progression of grid periods involving a non-integer scale factor close to 1, of size ≈ 1.42 [9,10]. A multi-period representation, as seen in the firing activity of the grid cells, is shown to have excellent representational and error-control properties [11,12] compared to unimodal tuning curves, assuming the existence of an appropriate decoder. More specifically, the grid cell code exhibits an exponential coding capacity with linearly many neurons in the presence of noise, a qualitatively different result from previously characterized population codes from the sensory and motor peripheries [12].

In this work, we generalize the conventional notion of shift-map codes [13–15] to include the grid cell code as an instance, where the difference between the grid cell code and the conventional shift-map codes is the choice of moduli. In particular, by choosing relatively prime integers as moduli, we extend the shift-map code to a novel form that resembles the grid cell code in the brain, and thus obtain a self-interleaving code (in the coding space). Given that this use of relatively prime integers shares a strong connection with redundant residue number systems (RRNS) [16–19,11], we refer to our codes as RRNS-map codes. Among our findings is the increased robustness of the RRNS-map code against noise compared to the conventional shift-map codes when side information (about the variable encoded) is present. Furthermore, the code is shown to be adaptive against noise and the quality of the side information. The contributions of this work include the following:

- We generalize the notion of shift-map codes to a new construction of analog code using relatively prime integers, referred to as the RRNS-map code.
- We design a set of RRNS-map codes whose codewords are well-separated. This novel structure allows the receiver to adaptively combine side information about the source for estimation, which offers an advantage over the traditional shift-map code.
- We offer a geometrical interpretation of the RRNS-map code.
- We provide a method to find RRNS-map codes with the desired properties. This method involves integer programming over relatively prime integers.
- As an extension, we show that the RRNS-map code is also adaptive for scenarios when only partial knowledge of the encoded variables is present.

The organization of the paper is as follows. Section 2 describes the system model. In Section 3, we briefly review the shift-map code and generalize it to include the RRNS-map code. In Section 4, the properties of the proposed construction are studied without side information. In Section 5, adaptive decoding of RRNS-map codes with side information is discussed and examples of good codes are provided. Extensions of the proposed code to different setups are discussed in Section 6. Concluding remarks are provided in Section 7.