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Pattern-based computing via sequential phase transitions in hierarchical mean field neuropercolation

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ABSTRACT

In this work, we describe operational principles of a pattern-based computing paradigm based on the neuropercolation model, which can be used as associative memory supporting sensory processing and pattern recognition. Neuropercolation extends the concept of phase transitions to interactive populations exhibiting frequent transients in their spatio-temporal dynamics, which can be viewed as manifestations of an asynchronous computer working with a sequence of meta-stable spatial patterns, in a bid to unravel the limitations of Turing computing principles. The model is motivated by the structural and dynamical properties of large-scale neural populations in the cerebral cortex and it implements basic building blocks of neurodynamics following the hierarchy of Freeman K-sets.

The introduced mean-field approximation allows rigorous mathematical analysis of the emergent dynamics, which is the major novel contribution of this work. Specifically, we derive exact conditions for the onset of non-zero background activity, for the transition from steady state to narrow-band (limit cycle) oscillations, and for the transition from narrow-band to broad-band (chaotic) dynamics. We describe an array of connected oscillators, which exhibits transient synchronization episodes manifesting meta-stable collective states. The corresponding meta-stable spatial amplitude patterns are destabilized by inputs or spontaneously and jump to another pattern, yielding a sequence of transient patterns. These patterns are shaped by the connections between the nodes modifiable through learning. The sequence of patterns manifest the steps of the computation, which embody the meaning of the input data in the context of the system past experiences.

1. Introduction

Mathematical approaches describing collective dynamics in brains are summarized here, including theory of dynamical systems and chaos, models of criticality, and network science and graph theory. Basic collective models consider Kuramoto's classical phase oscillator equations [1] to describe interactions in cortical networks [2,3]. Population models governed by neural mass equations manifest intermittent synchronization effects [4]. Complex spatio-temporal behaviors have been modeled using nonlinear ordinary and partial differential equations [5–8]. These approaches view brains as dynamical systems with evolving chaotic attractor landscapes [9–15]. With a focus on transient brain dynamics, principles of metastability have been introduced [16,17]. Chaotic itinerancy and Milnor attractors are mathematical models describing cognitive transients [18]. Frustrated chaos is yet another approach describing transient neural dynamics [19]. Metastable transients reflect

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sequential memory and they have been modeled successfully using heteroclinic cycles in competitive networks with excitatory and inhibitory interactions [20,21].

Synchronization has been observed between suitably connected oscillators, producing a range of collective behaviors [22]. Oscillators built of neural populations synchronize as a consequence of synaptic weight modifications [23,24]. In an ensemble of connected neural oscillators, each oscillator is driven by the coupling strengths of its neighboring oscillators. The onset of ensemble synchronization via a transition from desynchronization can be triggered by the increase of the coupling strengths beyond some critical values. Synchronization–desynchronization transitions are modeled as phase transitions [25] using the mathematical theory of random graphs and networks [26].

In network models, a vertex may represents an interacting neuron population, which can be either active of inactive. The state of each vertex is updated at discrete time steps according to some basic rules, depending on the states of its neighbors in the previous time step. A typical example is the majority rule, which states that the probability for the vertex to be active is high if most of its neighbors are active, while the probability of the vertex to be active is low if most of its neighbors are inactive. Majority rule is used in various popular models, such as Glauber dynamics, Toom's rule, Domany–Kinzel stochastic cellular automata, Bak–Tang–Wiesenfeld model, neuropercolation, and also in many voter models of interacting particles [27–33].

Neuropercolation is a family of probabilistic models based on the mathematical theory of random cellular automata on lattices and random graphs [33,34]. Neuropercolation develops equations for the probability distributions of macroscopic state variables generalizing percolation theory as an alternative to differential equations [35]. Neuropercolation results are interpreted in the context of recent experimental findings on the dynamics and structure of the cortex, indicating that brains operate at the edge of criticality, with phase transitions appearing intermittently, several times per second [36–38]. In the cinematic theory of cognition, brains compute with metastable coherent patterns as frames, intermittently interrupted by desynchronization episodes acting as the shutter [36]. Various aspects of criticality in brains were used as a blueprint to design and develop self-organized computing principles [39,40].

Basic principles of Freeman's neurodynamics serve as the foundation of a novel pattern-based computing approach [5,10].

- Interacting units become a population when they determine their activity collectively. Population is built by the excitatory connections among the nodes, which maintain a non-zero steady-state activity. *A population of spatially distributed nodes with this collective property is called Freeman KI set.*
- A sustained steady-state activity changes to prominent oscillations when an excitatory population is connected to an inhibitory population. The excitatory population increases the activity of units it projects to, while the inhibitory population decreases the activity of the (excitatory) population it is connected to. *An excitatory KI set connected with an inhibitory KI set is called KII.*
- Self-stabilized aperiodic (chaotic) background activity emerges as the result of negative and positive feedback between interconnected oscillators with excitatory and inhibitory populations. *Several interconnected KII oscillators form the KIII set.*
- Collective dynamics leads to the emergence of spatially distributed amplitude modulation (AM) patterns in KI, KII, and KIII sets. *Spatial AM patterns of population activity are metastable, i.e., they are mutually synchronized for a period of time but ultimately undergo destabilization through a large-scale desynchronization effect.* A graph as an ensemble of vertex populations can be desynchronized by one of the ensemble members or by external effects, which moves the graph from one metastable state to the other. As the flow of activity varies, metastable activity patterns emerge and disappear.
- The spatial AM patterns are attractor states created by modifiable graph edges through learning, and the patterns depend on the context and the history of the network. As a result of learning, broad-band basal activity transits to narrow-band oscillatory state with a metastable AM pattern corresponding to the meaning of that input. *The KIII network can be viewed as a dynamic associative memory operating through a self-organized sequence of amplitude patterns.*

This work starts with the description of a hierarchy of interacting populations with mean-field dynamics, including KI, KII, and KIII sets, motivated by Freeman's brain studies [10,41]. The introduced mean-field approximation allows rigorous mathematical analysis of the emergent dynamics, which is the major novel contribution of this work. Specifically, we derive exact conditions for the onset of non-zero background activity (KI); for the transitions between steady state and narrow-band limit cycle attractors (KII); and for the transition between narrow-band (limit cycle) and broad-band (chaotic) dynamics (KIII). We show that at the highest level of hierarchy (KIII), the graph dynamics enters a self-regulated critical regime, and the graph starts to transit between states with spatially synchronized patterns, interrupted by periods of intermittent desynchronization. Next, we describe a computational approach using neuropercolation models of interacting nodes operating at the edge of criticality. In our approach, metastable patterns are interpreted as manifestations of intermittent, context-dependent, emergent computational symbols, which are short-lived and disappear soon after they appear. Examples of transitions between narrow-band and broad-band oscillations are demonstrated in the presence of learnt input patterns. We conclude that synchronization–desynchronization transitions in KIII neuropercolation networks oscillating at criticality provide candidates for novel, context-dependent, dynamic computing paradigm. As compared to Turing machines, Neuropercolation Computation (NPerC) does not require a predefined high-level instruction set, rather the steps of computing are self-organized, based on large-scale collective dynamics. Finally, we indicate directions of ongoing and future research.