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Reeb graph based segmentation of articulated components of 3D digital objects

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ABSTRACT

Segmentation of the articulated components of digital objects by a fast and efficient algorithm is presented in this paper. Given a 3D object in the form of a triangulated mesh, the approach involves representation of the mesh in a topological space and relating quotient spaces, containing the topologically invariant sections of the object, to weighted Reeb graphs along each of *yz*-, *zx*-, and *xy*-planes. It is followed by segmentation of the Reeb graphs where the concept of exponential averaging for dynamic thresholding ensures natural segmentation with a relatively high degree of accuracy. The segmented quotient spaces corresponding to the three Reeb graphs are subsequently related to each other and transformed topologically to report the segmented object in an appropriate topological space. The segmentation is carried out in a framework of 3D grid so as to exploit the topological relation between the triangulated object and its tight isothetic cover, and hence involves efficient computation and storage. The accuracy of segmentation at different grid resolutions, its robustness w.r.t. rotation, and pose-invariance for a reasonable range of postures are demonstrated by experimental results on a variety of objects.

1. Introduction

Three-dimensional segmentation of digital objects is a well-studied concept and has been approached from various perspectives. Its widespread applications in shape analysis and understanding of peripheral topology of digital objects have contributed to its importance in various fields including 3D shape retrieval, texture matching, skeleton extraction, etc. Apart from geometric segmentation, several research works based on semantic-oriented segmentation have been reported. In [1], a hierarchical and pose-invariant segmentation is proposed where the mesh is initially transformed by multi-dimensional scaling. It is followed by extraction of feature points and core component by the use of spherical mirroring. Segmentation by randomized cuts, diffusion distance, and medial structures has been attempted [2,3], which results in hierarchical and pose-invariant segmentations. Morphological watersheds [4] and skeletonization have also been used as a tool for segmentation where shape diameter function deals with the volume of the object instead of surface attributes that alter with changes in pose and object topology [5]. Simultaneous separation of the articulated portions of a class of objects provided as aligned meshes is done in [6,7], and [8], without prior knowledge of the number of parts. The work in [7] uses the distance-dependent Chinese restaurant process (ddCRP) to maintain the spatial dependencies among the mesh triangles. On the other hand, a graph-theoretic approach based on the triangular faces of all the models in the set is used in [6], and an

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approach based on integer quadratic programming is used in [8] to bring about optimization over possible segmentations of individual shapes as well as multiple shapes. Segmentation based on topological maps and adjacency graphs has been proposed in [9] and that based on topological skeletons in [10], which takes into account some geometric properties of objects apart from their topology.

For all the mechanisms mentioned above, the input dataset is a triangulated mesh representing the digital object. If the input dataset is an unstructured three-dimensional point cloud, then methods involving random walks [11], piecewise intersection of planar faces [12], etc. are used for direct segmentation of the point cloud, and hence that of the object. Segmentation from different other types of 3D point clouds is reported in [13]. 3D segmentation has also been attempted for a given sequence of 2D images, by using graph cut optimization [14,15], or by optimizing a cost function based on a Conditional Random Field using dynamic graph cuts [16,17]. Other related techniques and some of their comparisons can be seen in [18–24].

Irrespective of the adopted technique, there always lies a chance of over-segmentation [25], which, at times, affects the accuracy of segmentation, and sometimes may also assist in deriving important information about the object [26]. In this paper,[1] we propose a novel method to obtain natural segmentation, which is fast, robust, rotationally invariant, pose-invariant, and free from skeletonization. The approach is based on the idea of representing the object surface as a topological space followed by an effective method of segmentation involving weighted Reeb graphs along each of *yz*-, *zx*-, and *xy*-planes. Natural segmentation with a relatively high degree of accuracy is achieved by setting the threshold of segmentation dynamically using exponential averaging. A topological relation between the given triangulated object and the 3D isothetic cover of the object [28,29] facilitates segmentation in the orthogonal domain with varying grid resolutions [30], thereby leading to efficient computation and storage.

Graph-theoretic analysis of 3D objects is a well-explored area of shape representation that is characterized by a structural description of the object based on its geometric and topological properties. Topological variation of manifolds, particularly for articulated objects, may be suitably encoded by Reeb graphs as it has a one-dimensional graph structure and is invariant to both global and local transformations [37,38]. In [36], decomposition of 3D objects is proposed that uses Reeb graph to extract structural and topological information of the object surface followed by refinement of part boundaries using curvature information. But unlike the current work where Reeb graph construction is based on orthogonal slicing information, in [36] Reeb graph is encoded with respect to the elevation of surface points. Identification of human body parts from an unorganized point cloud of scanned data has been reported in several works. A combination of global shape description and a local criteria of proximity regarding the relative positions of the body parts is used in [31]. There the scanned dataset is organized into a stack of slices and aligned with a predefined template for locating the different articulated sections of the human body. An improvement of this approach has been suggested in [35] where curvature analysis is used for segmentation. A further modification by Reeb graph is presented in [32,33] where the 3D point cloud is organized into level-sets or slices along the direction of the human body which are used to construct the Reeb graph for segmentation. However, there are differences between the work in [32,33] and the one proposed by us. In [32,33], the input dataset is a 3D point cloud which involves higher space complexity w.r.t. our algorithm in which a triangulated mesh is used as input. Secondly, the algorithm in [32,33] is applicable for only objects like human body that have a specific direction for height under standard posture. Though this shortcoming has been overcome in [34] where posture changes of human body have been accounted for by using geodesic distance, the approach is still restricted by a specific structure of the slices pertaining only to the human body. Our algorithm, on the other hand, is effective for a wide range of objects including the human body even when they are oriented in various directions. A brief comparison of the existing algorithms with the proposed algorithm is presented in Table 1.

The rest of the paper is organized as follows. In Sec. 2, a given digital object is represented as a Reeb graph by using topological means. An efficient algorithm for segmentation of the Reeb graph along the three coordinate planes is presented in Sec. 3. In Sec. 4, the three segmented Reeb graphs are related topologically to determine the final segmentation of the object into its articulated components. A number of experimental results and statistical information are presented in Sec. 5 that demonstrate the stability, robustness, and accuracy of segmentation under various conditions, displayed by the proposed method.

2. Representation of digital object by Reeb graph

We consider a 3D object A in the form of a triangulated surface, which is closed and orientable (2-manifold). On each edge of the resultant triangulation, exactly two triangles are incident. As the surface is closed, it is a compact surface without boundary; and as it is orientable, it can be traversed with a notion of positive rotation consistently assigned on it. With an aim to perform natural segmentation of objects using operations in the orthogonal domain, the triangulated mesh is associated with a 3D isothetic cover of the object [29,28]. The 3D isothetic cover $\overline{P}_{\mathbb{G}}(A)$ of a digital object is a 3D polytope of minimum volume defined w.r.t. an underlying grid $\mathbb{G}$ having surfaces parallel to the coordinate planes and circumscribing the entire object [29]. The grid $\mathbb{G}$ is represented as a set of *unit grid cubes* (UGCs), each of length g. A triangle t of the mesh corresponds to a UGC u if (i) $u \nsubseteq A$, (ii) t intersects at least one UGC-face $f_k \in u$, and (iii) at least one UGC-face $f_l \in u$ is

[1] A preliminary version appeared in [27].