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Reconstruction of convex polyominoes with a blocking component

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ABSTRACT

Several papers in the available literature tackled problems concerning convex polyominoes in discrete tomography. An interesting subclass consists of L-convex polyominoes, since the related reconstruction problem can have only a unique solution. On the other hand, recent studies have modeled an approach to reconstruct objects even in the case that some of the projections are unavailable, due to a particularly dense part of the scanned object, that we refer to as a blocking component. In this work we merge the two problems in order to obtain efficient reconstruction algorithms for convex and L-convex polyominoes, in case a blocking component is included.

1. Introduction

Discrete tomography deals with the reconstruction of discrete shapes by means of their projections along given directions. Often a priori assumptions are considered, in order to take into account constraints deriving from applications or to employ efficient reconstruction algorithms; these assumptions may also yield limitations on the number of solutions of a problem. These range from typical constraints, such as connection and convexity [4,26], boundedness [10,12], multicoloredness [3,22], tiling [15], to newer and different assumptions, such as skeletal properties [28], scheduling constraints [9], object based images [7] and Japanese puzzles [5]. Many studies in discrete tomography also examine properties of uniqueness, that is very relevant for applications; see for example [1,11,20,24,25,27,31]. A classical survey can be found in [29].

In this work we wish to match the a priori knowledge that the shape to be reconstructed is a convex polyomino with the assumption that some information in the available data is lacking, for instance some projections are not accessible. In applications, this can be caused by some obstruction which prevents a complete acquisition of the projections, due to a very dense part of the object that we call a blocking component. Polyominoes have been studied for a long time, and environments with blocked projections have been investigated, mainly heuristically, in applications. This area of research is known under the acronym MAR (metal artifact reduction), where strongly attenuating objects cause the missing data in projection vectors. A kind of variation of this problem also occurs when we have limited access to projection data since the probe source, or the detector, may not be able to physically access the entire space surrounding the object. Many medical applications may suffer from this problem, as in the case of dental fillings, surgical clips, orthopedic implants or digital breast tomosynthesis, and in many cases increasing the energy of the x-rays does not solve the problem, as reported in [17]. The usual approaches consist of modified reconstruction algorithms, such as projection completion methods and iterative

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procedures. In the first case, missing data are replaced by synthetic data, obtained by polynomial or linear interpolation. In the other approach, classical iterative algorithms, such as ART, are adapted, in order to manage the missing projections (see, for instance [17,19,23] and the related bibliography). On the other hand, exact algorithms for discrete tomography with a blocking component have been proposed only more recently in [6], allowing to explore a more theoretical view of the model. Merging these two lines of research seems to be a natural evolution of the state of the art.

The paper is organized as follows: in Section 2 we set the notation employed in the paper. Section 3 deals with the reconstruction algorithm for convex polyominoes. It is based on a previously existing procedure [16], but with several major modifications in order to adapt the approach when blocking components appear. Section 4 proposes an efficient reconstruction algorithm for L-convex polyominoes, and, in Subsection 4.1, we discuss the number of possible solutions of the problem. Section 5 is devoted to conclusions, with a few comments in view of possible future extensions.

2. Notation and preliminaries

First of all we wish to introduce the notion of *convex polyominoes*, also known in the literature as *hv-convex polyominoes*, since convexity refers to the horizontal and vertical directions. Polyominoes can be represented as discrete sets of cells, or, alternatively, in terms of binary matrices, which is the notation that we are adopting in the present paper.

A binary matrix P represents a convex polyomino if it satisfies the two following constraints:

- Connectedness: for any two entries of P with value 1, there is a path formed by horizontal and vertical steps connecting them and touching only other entries with value 1.
- Convexity in the horizontal and vertical direction: no row and no column of P contains the pattern $1, 0, \dots, 0, 1$.

In this paper figures represent polyominoes with the entry corresponding to $(1, 1)$ on the upper-left of the matrix. The input data of our problem are provided by the horizontal and vertical projections $H = (h_1, \dots, h_r)$ and $V = (v_1, \dots, v_c)$, where r, c are two integer numbers representing, respectively, the number of rows and columns of the polyomino to be reconstructed. We use the special value '?' to denote entries of H and V corresponding to unavailable projections. We assume that these positions are defined by four known indices B_1, B_2, B_3, B_4, where $h_{B_1} = h_{B_1+1} = \dots = h_{B_2} =?$ and $v_{B_3} = v_{B_3+1} = \dots = v_{B_4} =?$. The other entries of H and V shall be positive integers. The *sizes* of the blocking component are $B_h = B_2 - B_1 + 1$ and $B_v = B_4 - B_3 + 1$.

We also need the notion of *feet*. These are relevant positions of a polyomino which have been employed in several reconstruction algorithms. More precisely, a *west foot* is any entry $(i, 1) = 1$, an *east foot* is any entry $(i, c) = 1$, a *north foot* is any entry $(1, j) = 1$ and a *south foot* is any entry $(r, j) = 1$.

Now we define L-convex polyominoes, a subclass of convex polyominoes that exhibits interesting properties from the discrete tomography reconstruction point of view.

Definition 1. A convex polyomino is L-convex if, for every pair of entries $(i, j) = (i', j') = 1$ of the associated matrix P, at least one of the values $(i, j'), (i', j)$ is equal to 1.

The above definition differs from the one proposed in [13] (the work introducing convex polyominoes in a discrete tomography setting).[1] The two definitions are equivalent, but the one adopted here allows a simple and quick definition of the class, since it implies that any two 1s of P can be connected by a path with at most one change of direction (L-shaped), entirely included in the polyomino. A tomographic characterization of these polyominoes has been proposed in [13], where the following property was stated and proved. We say that a vector H (or V) is unimodal if there are no indices $i_1 < i_2 < i_3$ such that $h_{i_1} > h_{i_2}$ and $h_{i_2} < h_{i_3}$.

Property 1. *Suppose that $H = (h_1, \dots, h_r)$ and $V = (v_1, \dots, v_c)$ are horizontal and vertical projections vectors. Let $\overline{H}$ be the arrays obtained by non-increasingly rearranging the entries of H, and let $V^* = (v^*_1, \dots, v^*_r)$ be such that $v^*_j = |\{i \in \{1, \dots, c\} : v_i \geq j\}|$. Then, an L-convex polyomino satisfying H and V exists if and only if $\overline{H} = V^*$ and both H and V are unimodal.*

Note that $\overline{H} = V^*$ implies that there is only one binary matrix with projections H and V. Therefore, by using the projection data, we can easily apply Ryser's reconstruction algorithm to find this unique matrix, that hence must result to be an L-convex polyomino. Uniqueness is one of the properties that make L-convex polyominoes relevant, as determining when a problem has at most one solution is a widely studied problem in discrete tomography. In [21] *D-inscribable* lattice sets have been considered, being D a finite set of $k \geq 2$ lattice directions. Furthermore, in case D consists of the coordinate directions, D-inscribable lattice sets include L-convex polyominoes as a subclass, so that the reconstruction algorithm proposed in [21] also applies to such polyominoes.

[1] The notion of L-convexity has been employed in the literature also in different contexts, see for instance [30].