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Offline black and white bin packing

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ABSTRACT

We define and study a variant of bin packing called *unrestricted black and white bin packing*. Similarly to standard bin packing, a set of items of sizes in $[0, 1]$ are to be partitioned into subsets of total size at most 1, called bins. Items are of two types, called black and white, and the item types must alternate in each bin, that is, two items of the same type cannot be assigned consecutively into a bin. Thus, a subset of items of total size at most 1 can form a valid bin if and only if the absolute value of the difference between the numbers of black items and white items in the subset is at most 1.

We study this problem both with respect to the absolute and the asymptotic approximation ratios. We design a fast heuristic whose absolute approximation ratio is 2. We also design an APTAS and modify it into an AFPTAS. The APTAS can be used as an algorithm of absolute approximation ratio $\frac{3}{2}$, which is the best possible absolute approximation ratio for the problem unless $P = NP$.

1. Introduction

We study the offline bin packing problem, called *Black and White Bin Packing* (*BWBP*). As in standard bin packing, a set of items $I = \{1, \ldots, n\}$ is given, where item i is characterized by its size p_i, where $0 \le p_i \le 1$ holds[5], and the goal of the

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[5] A more standard assumption is $0 < p_i \le 1$, however in the problem studied here, zero sized items cannot be neglected, and thus we allow such items as well.

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bin packing problem is to pack them into the minimum number of unit capacity bins. This well-known problem [7,20,28] is NP-hard (see [18]), and has been widely studied.

In our problem the items are divided into two types or classes, i.e., every item is either black or white. In addition to the capacity constraint, we require that no two items of the same color are packed into a bin consecutively. For an item $i \in I$, we let c_i be its color. Specifically, if i is black, we let $c_i = 1$, and if i is white, then we let $c_i = -1$. In papers [2,1] the variant that we study here is called the *unrestricted offline* variant, i.e., the input is given as a set (rather than a sequence as in the online problem). These papers describe applications of the problem, and provide motivation for its study. Let $X \subseteq I$ be a subset of items. We let the color of X, $C(X)$, be $\sum_{i \in X} c_i$. A subset X of items, where $\sum_{i \in X} p_i \leq 1$, can form a valid bin if and only if $|C(X)| \leq 1$, and we let $s(X) = \sum_{i \in X} p_i$. For the subset X, if $C(X) = 1$, we say that X is black, if $C(X) = 0$, we say that X is neutral, and if $C(X) = -1$, we say that X is white. We use an additional enumeration of the items according to color. Let n_w be the number of white items, and let n_b be the number of black items (such that $n_w + n_b = n$). Let $w_1 \geq w_2 \geq \cdots \geq w_{n_w}$ be the sizes of white items, and let $b_1 \geq b_2 \geq \cdots \geq b_{n_b}$ be the sizes of black items. With a slight abuse of notation, we also denote the jth white item in this ordering (which has size w_j) by w_j, and similarly, we denote the kth black item in the ordering (which has size b_k) by b_k. We will assume without loss of generality that $n_b \geq n_w$ (these values can be easily computed in linear time, and the roles of white and black items can be reversed if necessary).

The absolute approximation ratio of an algorithm is the supremum ratio between the cost of the algorithm on the input I and the optimal cost for the same input, denoted by $OPT(I)$, that is, the minimum number of bins required to pack all items. To define the asymptotic approximation ratio, consider the function of N defined as the absolute approximation ratio limited to inputs for which the optimal cost is at least N. The asymptotic approximation ratio is the limit of the sequence of the absolute approximation ratios when N tends to infinity. An alternative (slightly stronger) definition states that an algorithm has an asymptotic approximation ratio of at most R if the cost of the algorithm for an input I is at most $R \cdot OPT(I) + C$, where $C \geq 0$ is an additive constant independent of the input. We will use the second definition, though our results are obviously valid for the first one as well. A polynomial time approximation scheme (PTAS) is a family of algorithms that for every $\varepsilon > 0$ contains an algorithm whose absolute approximation ratio is at most $1 + \varepsilon$. The running time must be polynomial in n, but not necessarily polynomial in $\frac{1}{\varepsilon}$. A fully polynomial time approximation scheme (FPTAS) is a PTAS where the running time is polynomial also in $\frac{1}{\varepsilon}$. If the approximation ratio is defined in the asymptotic sense, such schemes are called APTAS's and AFPTAS's, respectively.

We are interested in fast heuristics that achieve small absolute approximation ratios, and in the best possible approximation ratio (both in the absolute sense and the asymptotic sense). Similar to standard bin packing, a reduction from 3-Partition shows that the problem is NP-hard in the strong sense, and a reduction from Partition shows that the problem cannot be approximated within an absolute approximation ratio smaller than $\frac{3}{2}$ provided that $P \neq NP$ holds[6]. We show that this bound is achievable. We also design an AFPTAS. Thus, our results are the best possible (unless $P = NP$). The extended abstract [2] contained (in addition to results for the online variant, which were expanded into paper [1]) the APTAS and a fast heuristic of absolute approximation ratio $\frac{5}{2}$.

Standard bin packing is usually studied with respect to the asymptotic approximation ratio, but for the algorithms Next Fit [21] and First Fit [22,28] (defined in the body of the paper) it is known that their asymptotic approximation ratios, 2 and $\frac{17}{10}$, respectively, are actually also their absolute approximation ratios [21,11], and First Fit Decreasing (First Fit applied to a non-increasingly sorted input) has an absolute approximation ratio of $\frac{3}{2}$ [25], which is the best possible unless $P = NP$. Multiple variants of bin packing have been studied [8,9], and many of them have APTAS's and even AFPTAS's [17,23,24,5,3,27,12–16]. Specifically, an APTAS for standard bin packing was designed by Fernandez de la Vega and Lueker [17] and an AFPTAS was designed by Karmarkar and Karp [23]. A related variant is cardinality constrained bin packing, where a parameter k is given, such that no bin may contain more than k items. A generalization of this problem is vector bin packing, where every item is a d-dimensional vector with components in $[0, 1]$, and a bin capacity is an all-1 d-dimensional vector. The conditions of BWBP are however such that it is not a special case of vector bin packing. The cardinality constrained bin packing problem also has an APTAS and an AFPTAS [5,15]. A generalized variant of the online problem, called *colored bin packing* or *colorful bin packing*, was studied [10,4]. In this problem, an item has a color out of a possibly larger set of colors, and any two items packed consecutively in a bin must have distinct colors.

In Section 2 we present two fast heuristics. The first one has running time of $O(n \log n)$ and its absolute approximation ratio is 2. The second one is very simple and its running time is $O(n)$, but its absolute approximation ratio is 3. In Section 3 we present an FPTAS for an auxiliary knapsack problem which we define. This NP-hard problem is the knapsack variant of BWBP and is interesting in its own right, though the FPTAS for it is designed using standard methods (trimming and dynamic programming, see [19,29,6,26]). In Section 4, we present an exact algorithm for the case that there is a fixed number of item types (where the items are still given as a list of length n). The algorithm is based on an integer program, and we also show how to obtain a solution that can be computed more efficiently, though its cost can be larger than the cost of the solution obtained using the integer program by an additive term. Finally, in Section 5 we present our main results, the APTAS with an additive constant of 1, the algorithm of absolute approximation ratio $\frac{3}{2}$ and the AFPTAS, that build on the auxiliary problems in the previous sections. One idea of the approximation schemes is to partition items into

[6] In these reductions, a white item is created for every number in the input, and additionally, one black item of size zero is created together with the creation of every white item.