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Quell ☆

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ABSTRACT

We study the computational complexity of the puzzle Quell. The goal is to collect pearls by sliding a droplet of water over them in a map that is a two-dimensional grid. Each cell of the grid is either a free space (possibly with a pearl in it) or an obstacle. In each move, the droplet slides in one of the four directions to the maximal extent, through the free spaces of the map and collecting all pearls along the way, until it is stopped by an obstacle. We show that ANY-MOVES-ALL-PEARLS (deciding whether it is possible to collect all the pearls using any number of moves) can be solved in polynomial time. In contrast, both ANY-MOVES-MAX-PEARLS (finding the maximum number of pearls that can be collected using any number of moves) and MIN-MOVES-ALL-PEARLS (finding the minimum number of moves required to collect all the pearls) are APX-hard, although the corresponding decision problems are in FPT. We also present a simple 2-approximation for ANY-MOVES-MAX-PEARLS, and leave open the question whether MIN-MOVES-ALL-PEARLS admits a polynomial-time constant approximation.

1. Introduction

Quell is a popular puzzle developed by Fallen Tree Games.[1] The goal is to collect pearls by sliding a droplet of water over them in a map that is a two-dimensional grid. Each cell of the grid is either a free space (possibly with a pearl in it) or an obstacle. In each move, the droplet slides in one of the four directions to the maximal extent, through the free spaces of the map and collecting all pearls along the way, until it is stopped by an obstacle. Refer to Fig. 1. Throughout the paper, we focus on the *basic version* of Quell where the map contains no special objects, and moreover the pearls and the droplet are in a bounded and connected region of free spaces surrounded by obstacles. We say that the map is *simple* if the region is simply connected, that is, the region has no holes.

The maximal sliding movement of the droplet in Quell naturally models the movement of a robot with limited sensing capabilities navigating in an unfamiliar environment. While numerous games and puzzles have been rigorously studied in terms of their computational complexities, and many of them involve moving objects in a grid map (for example, Sokoban [8]), only a few adopt the maximal sliding model for the movement of the objects. They include Andrea Gilbert's Tilt Maze,[2] Lunar Lockout [15,12], ATOMIX [16], Randolph's Robot Game [10], PushPush and PushPushPush [9,14], and Pokémon [2].

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1 http://www.fallentreegames.com.

2 http://www.logicmazes.com/tilt.html, http://www.clickmazes.com/indext.htm.

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0304-3975/

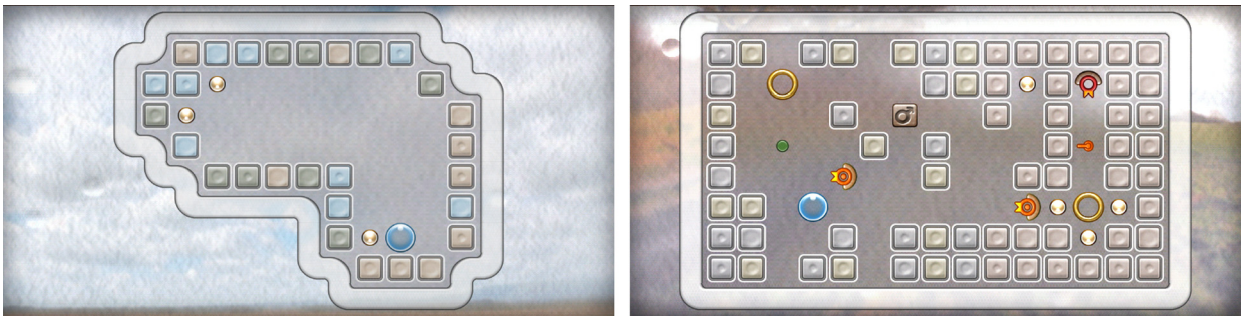

Fig. 1. Screenshots of two maps of Quell. Obstacles are depicted as square blocks, pearls as small yellow orbs, and the water droplet as a larger blue ball. Left: a simple map for the basic version of Quell. The three pearls can be collected with seven moves: *left, up, left, down, right, up, left*. Right: a more complex map with gaps in the boundary and with special objects. After the droplet exits through a gap in the boundary, it enters from the gap on the opposite side. After the droplet enters one golden ring, it is teleported to the other ring and keeps sliding in the same direction. The one-pass gate, shown as a small dark green dot (below the left ring), turns into an obstacle after the droplet passes through it. The block with the ♂ symbol can be pushed by the droplet. The droplet is destroyed when it slides onto a spike; the switch (above the right ring) changes the directions of the spikes (there are three spikes here, one near the droplet, one to the left of the right ring, and one above the switch) when the droplet passes through it. The four pearls can be collected with 26 moves. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

The basic version of Quell is equivalent to the multi-goal version of Tilt Maze (Tilt Maze predates Quell by about ten years). Unlike Quell, in which the droplet is the only moving object, Lunar Lockout, ATOMIX, and Randolph's Robot Game have swarms of moving robots. In PushPush and PushPushPush, there is only one agent that moves by itself, but the agent can push other blocks, so still there are multiple moving objects. In Pokémon, the agent can trigger the movements of multiple trainers by stepping into their lines of sight.

The challenging aspects of puzzles such as Lunar Lockout are mostly related to the complicated interaction among the multiple moving objects. The corresponding computational problems often turn out to be NP-hard and in PSPACE, and the central question regarding their computational complexities is whether they are PSPACE-hard or in NP. Indeed, in the original, purest version of Lunar Lockout, the whole space consists of only moving robots and nothing else, and whether the problem is PSPACE-hard or in NP has remained open for more than a decade.

On the other hand, the difficulty of the basic version of Quell (and of the multi-goal version of Tilt Maze) is mainly due to the contrast between the severe restriction of the maximal sliding movement and the complexity of the geometric environment. The three decision problems that we study in this paper are easily shown to be in NP; the central question is whether they are NP-hard or in P.

The most fundamental problem about Quell is the following decision problem:

- ANY-MOVES-ALL-PEARLS: decide whether it is possible to collect all the pearls using any number of moves.

The following two parameterized decision problems also arise naturally:

- ANY-MOVES-k-PEARLS: decide whether at least k pearls can be collected using any number of moves;
- k-MOVES-ALL-PEARLS: decide whether all the pearls can be collected using at most k moves.

We show that ANY-MOVES-ALL-PEARLS is in P and, in contrast, both ANY-MOVES-k-PEARLS and k-MOVES-ALL-PEARLS are NP-complete and are in FPT with parameter k (recall that FPT is the class of fixed-parameter tractable problems).

The two parameterized decision problems correspond to the following two optimization problems:

- ANY-MOVES-MAX-PEARLS: determine the maximum number of pearls that can be collected using any number of moves.
- MIN-MOVES-ALL-PEARLS: determine the minimum number of moves required to collect all the pearls.

For MIN-MOVES-ALL-PEARLS, if it is impossible to collect all the pearls using any number of moves, then the minimum number of moves is defined to be infinity. Since ANY-MOVES-ALL-PEARLS is in P, we can detect such instances in polynomial time. Throughout the paper, we exclude such instances for MIN-MOVES-ALL-PEARLS to avoid technical difficulty in defining the approximation ratio.

We show that both ANY-MOVES-MAX-PEARLS and MIN-MOVES-ALL-PEARLS are APX-hard (recall that APX-hard problems are unlikely to admit polynomial-time approximation schemes), and give a simple 2-approximation for ANY-MOVES-MAX-PEARLS. For MIN-MOVES-ALL-PEARLS, however, we are unable to find any constant approximation, even in simple maps.

The rest of this paper is organized as follows: