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Connected surveillance game ☆

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ABSTRACT

The *surveillance game* [4] models the problem of web-page prefetching as a pursuit evasion game played on a graph. This two-player game is played turn-by-turn. The first player, called the *observer*, can mark a fixed amount of vertices at each turn. The second one controls a *surfer* that stands at vertices of the graph and can slide along edges. The surfer starts at some initially marked vertex of the graph, its objective is to reach an unmarked node before all nodes of the graph are marked. The *surveillance number* $\text{sn}(G)$ of a graph G is the minimum amount of nodes that the observer has to mark at each turn ensuring it wins against any surfer in G. Fomin et al. also defined the *connected surveillance game* where the observer must ensure that marked nodes always induce a connected subgraph. They ask what is the cost of connectivity, i.e., is there a constant $c > 0$ such that the ratio between the *connected surveillance number* $\text{csn}(G)$ and $\text{sn}(G)$ is at most c for any graph G. It is straightforward to show that $\text{csn}(G) \leq \Delta\,\text{sn}(G)$ for any graph G with maximum degree Δ. Moreover, it has been shown that there are graphs G for which $\text{csn}(G) = \text{sn}(G) + 1$. In this paper, we investigate the question of the cost of the connectivity.

We first provide new non-trivial upper and lower bounds for the cost of connectivity in the surveillance game. More precisely, we present a family of graphs G such that $\text{csn}(G) > \text{sn}(G) + 1$. Moreover, we prove that $\text{csn}(G) \leq \sqrt{\text{sn}(G)n}$ for any n-node graph G. While the gap between these bounds remains huge, it seems difficult to reduce it. We then define the *online surveillance game* where the observer has no *a priori* knowledge of the graph topology and discovers it little-by-little. This variant, which fits better the prefetching motivation, is a restriction of the connected variant. Unfortunately, we show that no algorithm for solving the online surveillance game has competitive ratio better than $\Omega(\Delta)$. That is, while interesting, this variant does not help to obtain better upper bounds for the connected variant. We finally answer an open question [4] by proving that deciding if the surveillance number of a digraph with maximum degree 6 is at most 2 is NP-hard.

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1. Introduction

In this paper, we study two variants of the *surveillance game* introduced in [4]. This two-player game involves one Player moving a mobile agent, called *surfer*, along the edges of a graph, while a second Player, called *observer*, marks the vertices of the graph. The surfer wins if it manages to reach an unmarked vertex. The observer wins otherwise.

Surveillance game. More formally, let $G = (V, E)$ be an undirected simple n-node graph, $v_0 \in V$, and $k \in \mathbb{N}^*$. Initially, the surfer stands at v_0 which is marked and all other nodes are not marked. Then, turn-by-turn, the observer first marks k unmarked vertices and then the surfer may move to a neighbour of its current position. Once a node has been marked, it remains marked until the end of the game. The surfer wins if, at some step, it reaches an unmarked vertex; and the observer wins otherwise. Note that the game lasts at most $\lceil \frac{n}{k} \rceil$ turns. When the game is played on a directed graph, the surfer has to follow arcs when it moves [4]. A *k-strategy for the observer from* v_0, or simply a *k-strategy from* v_0, is a function $\sigma : V \times 2^V \to 2^V$ that assigns the set $\sigma(v, M) \subseteq V$ of vertices, $|\sigma(v, M)| \leq k$, that the observer should mark in the *configuration* (v, M), where $M \subseteq V$, $v_0 \in M$, is the set of already marked vertices and $v \in M$ is the current position of the surfer. We emphasize that σ depends implicitly on the graph G, i.e., it is based on the full knowledge of G. A k-strategy from v_0 is *winning* if it allows the observer to win whatever be the sequence of moves of the surfer starting in v_0. The *surveillance number* of a graph G with initial node v_0, denoted by $\mathrm{sn}(G, v_0)$, is the smallest k such that there exists a winning k-strategy starting from v_0.

Let us define some notations used in the paper. Let Δ be the maximum degree of the nodes in G and, for any $v \in V$, let $N(v)$ be the set of neighbours of v. More generally, the neighbourhood $N(F)$ of a set $F \subseteq V$ is the subset of vertices of $V \setminus F$ which have a neighbour in F. Moreover, we define the closed neighbourhood of a set F as $N[F] = N(F) \cup F$.

As an example, let us consider the following *basic strategy*: let $\sigma_{\mathcal{B}}$ be the strategy defined by $\sigma_{\mathcal{B}}(v, M) = N(v) \setminus M$ for any $M \subseteq V$, $v_0 \in M$, and $v \in M$. Intuitively, the basic strategy $\sigma_{\mathcal{B}}$ asks the observer to mark all unmarked neighbours of the current position of the surfer. It is straightforward, and it was already shown in [4], that $\sigma_{\mathcal{B}}$ is a winning strategy for any $v_0 \in V$ and it easily implies that $\mathrm{sn}(G, v_0) \leq \max\{|N(v_0)|, \Delta - 1\}$.

Web-page prefetching, connected and online variants. The surveillance game has been introduced because it models the web-page prefetching problem. This problem can be stated as follows. A web-surfer is following the hyperlinks in the digraph of the web. The web-browser aims at downloading the web-pages before the web-surfer accesses it. The number of web-pages that the browser may download before the web-surfer accesses another web-page is limited due to bandwidth constraints. Therefore, designing efficient strategies for the surveillance game would allow to preserve bandwidth while, at the same time, avoiding the waiting time for the download of the web-page the web-surfer wants to access.

By nature of the web-page prefetching problem, in particular because of the huge size of the web digraph, it is not realistic to assume that a strategy may mark any node of the network, even nodes that are "far" from the current position of the surfer. For this reason, [4] defines the *connected* variant of the surveillance game. A strategy σ is said *connected* if $\sigma(v, M) \cup M$ induces a connected subgraph of G for any M, $v_0 \in M \subseteq V(G)$. Note that the basic strategy $\sigma_{\mathcal{B}}$ is connected. The *connected surveillance number* of a graph G with initial node v_0, denoted by $\mathrm{csn}(G, v_0)$, is the smallest k such that there exists a winning connected k-strategy starting from v_0. By definition, $\mathrm{csn}(G, v_0) \geq \mathrm{sn}(G, v_0)$ for any graph G and $v_0 \in V(G)$. In [4], it is shown that there are graphs G and $v_0 \in V(G)$ such that $\mathrm{csn}(G, v_0) = \mathrm{sn}(G, v_0) + 1$. Only the trivial upper bound $\mathrm{csn}(G, v_0) \leq \Delta\, \mathrm{sn}(G, v_0)$ is known and a natural question is how big the gap between $\mathrm{csn}(G, v_0)$ and $\mathrm{sn}(G, v_0)$ may be [4]. This paper provides a partial answer to this question.

Still the connected surveillance game seems unrealistic since the web-browser cannot be asked to have the full knowledge of the web digraph. For this reason, we define the *online surveillance game*. In this game, the observer discovers the considered graph while marking its nodes. That is, initially, the observer only knows the starting node v_0 and its neighbours. After the observer has marked the subset M of nodes, it knows M and the vertices that have a neighbour in M and the next set of vertices to be marked depends only on this knowledge, i.e., the nodes at distance at least two from M are unknown. In other words, an *online strategy* is based on the current position of the surfer, the set of already marked nodes and knowing only the subgraph H of the marked nodes and their neighbours (a more formal definition is postponed to Section 3). By definition, the next nodes marked by such a strategy must be known, i.e., adjacent to an already marked vertex. Therefore, an online strategy is connected. We are interested in the competitive ratio of winning online strategies. The competitive ratio $\rho(\mathcal{S})$ of a winning online strategy $\mathcal{S}$ is defined as $\rho(\mathcal{S}) = \max_{G, v_0 \in V(G)} \frac{\mathcal{S}(G, v_0)}{\mathrm{sn}(G, v_0)}$, where $\mathcal{S}(G, v_0)$ denotes the maximum number of vertices marked by $\mathcal{S}$ in G at each turn, when the surfer starts in v_0. Note that, because any online winning strategy $\mathcal{S}$ is connected, $\mathrm{csn}(G, v_0) \leq \rho(\mathcal{S})\, \mathrm{sn}(G, v_0)$ for any graph G and $v_0 \in V(G)$.

1.1. Related work

The surveillance game has mainly been studied in the computational complexity point of view. It is shown that the problem of computing the surveillance number is NP-hard in split graphs [4,5]. Moreover, deciding whether the surveillance number is at most 2 is NP-hard in chordal graphs and deciding whether the surveillance number is at most 4 is PSPACE-complete. Polynomial-time algorithms that compute the surveillance number in trees and interval graphs are designed in [4]. All previous results also hold for the connected surveillance number. Finally, it is shown that, for any graph G and $v_0 \in V(G)$, $\max \lceil \frac{|N[S]|-1}{|S|} \rceil \leq \mathrm{sn}(G, v_0) \leq \mathrm{csn}(G, v_0)$ where the maximum is taken over every subset $S \subseteq V(G)$ inducing