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Relating the extra connectivity and the conditional diagnosability of regular graphs under the comparison model

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ABSTRACT

Extra connectivity and conditional diagnosability are two crucial subjects for a multiprocessor system's ability to tolerate and diagnose faulty processors. The extra connectivity and the conditional diagnosability of many well-known multiprocessor systems have been widely investigated. In this paper, the relationship between the extra connectivity and the conditional diagnosability of regular graphs is explored. We establish that the conditional diagnosability under the comparison model is equal to the 2-extra connectivity. Finally, we give empirical analysis on the extra connectivity and conditional diagnosability of some graphs by our proposed relationship.

1. Introduction

The *extra connectivity* is an effective measure of the reliability of a multiprocessor network. The *conditional diagnosability* has also played an important role in measuring the reliability of a multiprocessor system.

The *extra connectivity* was investigated by Fàbrega and Fiol [7], which is an important indicator of the robustness of a multiprocessor system in the presence of failing processors and overcomes the shortcomings for the *connectivity* [10]. The connectivity [10] tacitly assumes that all vertices adjacent to the same vertex could fail at the same time, but that is almost impossible in practical network applications. Consequently, the classical connectivity is not suitable for large-scale processing systems. The extra connectivity of various classes of graphs have been studied in recent years [3,4,13,18,23,29,30,32,35,38].

The ability to identify all faulty processors in a multiprocessor system is known as *system-level diagnosis*. Several system-level diagnosis models have been proposed for till now. One important model, namely the *PMC* (*Preparata, Metze, and Chien*) *model*, proposed by Preparata, Metze, and Chien [26], assumes that each vertex can only test its neighboring vertices, and the test results are either "faulty" or "fault-free". Another major diagnosis approach, proposed by Malek and Maeng [24,25], is called the *comparison model*. This diagnosis model assumes that each vertex can test its neighboring vertices by sending the same input to each pair of its distinct neighbors and then compare their responses. After a system has been diagnosed, identified faulty vertices are replaced with fault-free vertices.

The *conditional diagnosability* was proposed by Lai et al. [17] to better reflect the system's self-diagnostic capability under more practical assumptions, which overcomes the shortcomings for the *diagnosability* [8,9]. The classical diagnosability [8,9] of a network is quite small, because it imposes no conditions on the distribution pattern of faults. The conditional diagnosability is widely accepted as a new measure of diagnosability by assuming that any set of faulty vertices cannot

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contain all neighboring vertices of any vertex in a multiprocessor system. The conditional diagnosability of various classes of systems under the comparison model [6,12,14–16,20,31,33,34,36] have been extensively studied in recent years.

Based on the importance of the extra connectivity and the conditional diagnosability, our goal is to propose the relationship between extra connectivity and conditional diagnosability of regular graphs with some basic conditions. It is motivated by the recent researches on the extra connectivity and conditional diagnosability of some graphs, including hypercubes [30], bijective connection networks (BC networks) [29], alternating group graphs [11,23], dual-cubes [35] and Split-Star Networks [21,22].

The major contributions of this paper are as follows:

- We propose the fault tolerant properties of regular graphs with some basic conditions.
- We establish that the conditional diagnosability under the comparison model is equal to the 2-extra connectivity of regular graphs.
- We give empirical analysis on the extra connectivity and conditional diagnosability of star graphs by our proposed relationship, and show that the 2-extra connectivity of S_n ($n \geq 10$) is $3n-7$.
- We give empirical analysis on the extra connectivity and conditional diagnosability of the Cayley graphs generated by transposition trees by our proposed relationship, and show that the 2-extra connectivity of Cayley graphs generated by transposition trees ($n \geq 10$) is $3n-8$.

Organization. The remainder of this paper is organized as follows. Section 2 introduces terminology and notations that will be used throughout this paper. Section 3 proposes the fault tolerant properties of regular graphs with some basic conditions. Section 4 establishes the relationship between the extra connectivity and the conditional diagnosability. Section 5 gives empirical analysis. Section 6 concludes the paper.

2. Preliminaries

2.1. Terminology and notations

A graph $G = (V(G), E(G))$ is often applied to represent the topology of a multiprocessor system, where $V(G)$ is the *vertex-set* of G, elements in which are called vertices of G; $E(G)$ is the *edge-set* of G, elements in which are called edges of G. Let $|V(G)|$ be the order of G and $|E(G)|$ be the size of G. For notations and terminology not defined here please refer to [28].

Two vertices corresponding an edge are called the *end-vertices* of the edge. If at least one end-vertex of an edge is faulty, the edge is said to be *faulty*; otherwise, the edge is said to be *fault-free*. The *distance* between vertex u and vertex v, denoted by $d(u,v)$, is the length of a *shortest path* between u and v in G. If the length of a path (resp. cycle) of G is n, then we call the path (resp. cycle) as an *n-path* (resp. *n-cycle*). The *girth* of G, denoted by $g(G)$, is the minimum length of cycles in G.

Let S be a subset of $V(G)$, whose size is denoted as $|S|$. The *induced subgraph* by S, denoted by $G[S]$, is a subgraph of G whose vertex-set is S and whose edge-set $E(G[S]) = \{uv \mid uv \in E(G), u, v \in S\}$. Let $G_1, G_2, \ldots, G_m$ be m subgraphs of G, we set

$$\cup_{i=1}^{m} G_i = G[\cup_{i=1}^{m} V(G_i)] \text{ and } \cap_{i=1}^{m} G_i = G[\cap_{i=1}^{m} V(G_i)].$$

For any subset F of $V(G)$, the notation $G - F$ denotes a graph obtained by removing all vertices in F from G and deleting those edges with at least one end-vertex in F, simultaneously. Let M and N be any two distinct subsets of $V(G)$. The *symmetric difference set* of M and N, denoted by $M \triangle N$, is the union of $M - N$ and $N - M$, i.e., $M \triangle N = (M - N) \cup (N - M) = \{x \mid x \in M \cup N, x \notin M \cap N\}$. The cross edges between M and N, denoted by $E[M, N]$, is the set of all edges between M and N. For any vertex u of a graph $G = (V(G), E(G))$, the *neighborhood* of u in G, denoted by $N_G(u)$, is defined as a set of all vertices which are adjacent to u, i.e., $N_G(u) = \{v \in V(G) \mid uv \in E(G)\}$. We define

$$N_G(S) = \{v \in V(G) - S \mid \exists u \in S, uv \in E(G)\} = \bigcup_{u \in S} N_G(u) - S.$$

Let $N_G[u] = N_G(u) \cup \{u\}$ be the *closed neighborhood* of u in G and $N_G[S] = N_G(S) \cup S$. When G is clear from the context, we use $N(v)$, $N(S)$, $N[v]$ and $N[S]$ to replace $N_G(v)$, $N_G(S)$, $N_G[v]$ and $N_G[S]$, respectively. We also denote, by $|N(u)|$, the degree $d(u)$ of u. Let $\Delta(G)$ (resp. $\delta(G)$) refer to the *maximum (resp. minimum) degree* of vertices in G. For any one subset $V' \subset V(G)$, the set of *private neighbors* of one vertex $v \in V'$, denoted by $PN_{V'}(v)$, is the set of those neighbors of v which are not shared by other vertices in V' and are not themselves in V', i.e., $PN_{V'}(v) = N(v) - (N(V' - \{v\}) \cup V')$. For any two vertices u and v in G, let $cn(u,v)$ be the number of common neighbors of u and v. Let $cn(G) = \max\{cn(u,v) \mid u, v \in V(G)\}$. If a graph G such that $cn(u,v) \leq 2$ for any $uv \notin E(G)$ and $cn(u,v) \leq 1$ for any $uv \in E(G)$, then the graph G is called as a *2-cn-graph*.

A subset $F \subset V(G)$ is called a *fault-set* if each vertex in F is faulty. A subset $F \subset V(G)$ is called a *conditional fault-set*, if all vertices of set F are faulty and $N(v) \nsubseteq F$ for any $v \in V(G)$. We say that (F_1, F_2) is a *conditional fault-pair*, when F_1 and F_2 are conditional fault-sets.