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Solving the Many to Many assignment problem by improving the Kuhn–Munkres algorithm with backtracking

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A B S T R A C T

The Many to Many (M–M) assignment problem is an important open problem where one task is assigned to many, but different, agents and one agent may undertake many, but different, tasks. The Kuhn–Munkres (K–M) algorithm is a famous and traditional process used in dealing with assignment problems. In this paper, we propose a solution to the M–M assignment problem by improving the K–M algorithm with backtracking (KM_B). To demonstrate the solution's suitability, we prove that the proposed KM_B algorithm is valid and that the worst time complexity of the KM_B algorithm is $O((\sum L^a[i])^3)$, where $L^a[i]$ denotes the maximum number of tasks that can be assigned to agent i. After that, we discuss several critical problems related to the algorithm and provide the necessary and sufficient conditions of solving the M–M assignment problem. Finally, we demonstrate, through experimentation, the validity, practicality and efficiency of the KM_B algorithm.

1. Introduction

The assignment problem is one of the fundamental combinatorial optimization problems in the branch of optimization or operations research of mathematics. The assignment problem considers that there are agents available to undertake certain tasks. Any agent can be assigned to perform any task based on agents' abilities on tasks. In collaboration, agent-task assignment aims at the highest performance based on the agents' performances [1]. Such a problem is similar to finding a maximum weight matching (or minimum weight perfect matching) in a weighted bipartite graph. A traditionally used tool is called the Hungarian method (also known as the Kuhn–Munkres algorithm) [2,3].

The Kuhn–Munkres (K–M) algorithm always requires that a task be assigned to exactly one agent who undertakes exactly one task. This constraint limits the KM algorithm to solving the One to One (1–1) assignment problem. With respect to the KM algorithm, the M–M assignment problem remains open [4].

In contrast to 1–1 assignment, the Many to Many (M–M) assignment problem allows one task to be undertaken by many, but different, agents and allows one agent to perform many, but different, tasks. For example, in a team, different people may be assigned to one or more tasks without repetition. In services, different servers may provide different services. Note that, if tasks and agents can be assigned repeatedly, it is not novel within the processing scope of the K–M Algorithm.

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The M–M assignment problem is common and important. In this paper, we improve the KM algorithm to solve the M–M assignment problem by introducing backtrack processes. The improved version is called the Kuhn–Munkres algorithm with Backtracking (simplified as the KM_B algorithm). To demonstrate that the proposed algorithm is correct and efficient, we prove that the KM_B algorithm is valid and the worst time complexity of the KM_B algorithm is $O((\sum L^a[i])^3)$, where $L^a[i]$ denotes the maximum number of tasks that can be assigned to agent i. After that, we discuss several critical problems with the KM_B algorithm and provide the conditions necessary, and sufficient, for the solution of the M–M assignment problem. Finally, we illustrate the validity and efficiency of the KM_B algorithm through simulations, which also verify the algorithm's practicality.

2. Literature review and preliminaries

The One to Many (1–M) assignment (Called Group Role Assignment (GRA) in [1]) is itself a complex problem where the exhaustion algorithm has an exponential increase in complexity. In this field, an efficient algorithm was developed by using the Hungarian algorithm (also called Kuhn–Munkres algorithm [2,3]). The influence of the K–M algorithm on combinatorial optimization is fundamental, where it became the prototype of a great number of algorithms in the areas of network flows, matroids, and the matching theory [5].

There are many revised and enhanced versions of the KM algorithm. For example, Bourgerois and Lassalle [6] develop an extension of this algorithm that permits a solution for rectangular matrices. Toroslu and Üçoluk [7] introduce the incremental assignment problem and propose an $O(|V|^2)$ algorithm for the problem, where $|V|$ is the size of a partition in a bipartite graph. Bertsekas and Castañon [8] discuss the parallel implementation of the auction algorithm for the classical assignment problem. Brogan [9] enhances the algorithm for ranked assignments with applications to multi-object tracking. The *k-cardinality assignment problem*, introduced by Dell'Amico and Martello [10,11], asks one to assign exactly k rows to k different columns so that the sum of the corresponding costs is the minimum. Linear bottleneck assignment problems were introduced by Fulkerson, Glicksberg and Gross [10,12] and occur, e.g., in connection with assigning jobs to parallel machines, so as to minimize the latest completion time. Grygiel [10,13] investigates the Sum-k assignment problem within an algebraic framework and designs an $O(n^5)$ algorithm in the case of real cost coefficients. Our previous work [1] extends the KM algorithm to solve the GRA (or 1–M assignment) problem. To the author's knowledge, there is a lack of fundamental and comprehensive research on the M–M assignment problem.

In engineering applications, related research work focuses on role assignment for agents in multi-agent systems [14–16] or nodes in networked systems [17]. Dastani et al. [14] present research on the determination of conditions under which an agent can adopt a role and what it means for the agent to perform a role. Durfee et al. [18] propose a new formulation of the team formation by modelling the assignment and scheduling of expert teams as a hybrid scheduling problem. Shen et al. [19] propose a multi-criteria assessment model capable of evaluating the suitability of individual workers for a specified task according to their capabilities, social relationships, and existing tasks. Stone and Veloso [20] introduce periodic team synchronization (PTS) domains as time-critical environments in which agents act autonomously. Vail and Veloso [21] extend Stone and Veloso's work [20] in role assignment and coordination in multi-robot systems, especially in highly dynamic tasks. Choi and Bahk [22] use the KM algorithm to develop a heuristic limited-matching scheduling algorithm with the partial channel feedback in OFDMA (orthogonal frequency division multiple access) and MIMO (multiple input multiple output) systems. Emms, Wilson and Hancock [23] apply the KM algorithm to recover a correspondence mapping between the graphs. Huang, Xu and Cham [24] formulate an integer programming problem to compute the distances from one probe image to all the gallery images belonging to a certain class, in which any feature of the probe image can be matched to only one feature from one of the gallery images. It is obviously that almost all of these researches are limited by instances of the 1–1 assignment problem.

Although the research above indicates a strong need to fundamentally investigate the M–M assignment problem, it is still an open problem with respect to the traditional K–M algorithm.

2.1. Problem description

To aid in the description of the M–M assignment problem, we borrow some symbols from the Environments-Classes, Agents, Roles, Groups and Objects (E-CARGO) model as follows [1], where we assume that there are n tasks and m agents in the discussions:

Definition 1 *(Task range vector).* A *task range vector* L is a vector of n tasks, where $L[j]$ denotes that quantity of task j must be assigned ($0 \le j < n$).

Definition 2 *(Ability limit vector of agent).* An *ability limit vector* of m agents is L^a, where $L^a[i]$ denotes that how many tasks can be assigned to agent i at most ($0 \le i < m$).

Definition 3 *(Performance matrix).* A performance *matrix* Q is an $m \times n$ matrix, where $Q[i, j] \in [0, 1]$ expresses the performance value of agent i ($0 \le i < m$) for task j ($0 \le j < n$). $Q[i, j] = 0$ indicates the lowest value and 1 the highest.